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User's Manual for RTRAJ: A Trajectory Estimation and Simulation Program

R. H. Frick

A Project AIR FORCE report
prepared for the
United States Air Force

Rand
SANTA MONICA, CA. 90406

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PREFACE

This report presents a user's manual of operating instructions for the "RTRAJ" computer program, which has been under development and in use at The Rand Corporation for several years. (RTRAJ takes its name from "Rand trajectory program.") The function of the program is to provide an estimate of the accuracy with which the motion of a vehicle can be determined on the basis of either inertial guidance measurements or external tracking, or both.

The RTRAJ program was developed under the Future Strategic Aerospace Force Requirements task of Project AIR FORCE (formerly Project RAND). It should be useful to analysts in the fields of ballistic missile accuracy, range instrumentation, and navigational satellites.

SUMMARY

This report presents the analytical background and the operating instructions for a computer program, RTRAJ, developed at The Rand Corporation. The purpose of the program is to provide a best estimate of the performance of a moving vehicle as described by the estimated deviation from its nominal performance, as well as the precision with which this deviation is known. The modeled parameters whose deviations characterize the vehicle performance include the vehicle position and velocity, and may include the error sensitivity coefficients associated with an inertial measuring unit (IMU). Performance is determined by measurements with an onboard IMU or external trackers, or both. In the absence of any external tracking measurements, the position and velocity indicated by the IMU and the nominal values of the IMU parameters are the best and only estimates of vehicle performance. The precision of the position and velocity estimates is correlated with the a priori standard deviations assigned to the IMU parameters and varies as the vehicle moves along its path, whereas the standard deviations of the IMU parameters themselves are invariant.

If external tracking measurements of known precision are made in conjunction with the IMU, this additional information can be used to modify the performance estimate provided by the IMU by means of a sequential Kalman filter, which produces an improved estimate of the vehicle position and velocity as well as of the IMU parameters. At the same time, the precision of these estimates is also redetermined. In the event that no IMU is used, the external measurements are used to update the a priori estimates of performance and precision. In the absence of any unmodeled errors, the best estimate of the vehicle performance should approach its actual performance with increasing precision as the number of external measurements increases. It is possible, however, to introduce unmodeled errors in the form of unknown tracker position and velocity biases as well as biases in the measured quantities. With such unmodeled errors, the estimated vehicle performance deviates from the actual performance.

The trackers used to make the external measurements can be located on the ground, on aircraft, on satellites, or on the vehicle (as an altimeter), and can measure any or all of the following quantities: range, range rate, azimuth angle, elevation angle, hyperbolic range, and hyperbolic range rate.

ACKNOWLEDGMENTS

Several Rand staff members have been instrumental in the development of the RTRAJ program. They include R. L. Mobley, who wrote the original version; R. P. Castro, who incorporated the inertial guidance computations; and Eric Olson, who added the hyperbolic range and range rate measurements. The author would like to acknowledge their assistance in translating the program from the original FORTRAN into the language of the present report.

CONTENTS

FIGURES

SYMBOLS

[A]	Matrix transformation between [B] and [H].
$[A_{IPI}]$	Matrix transformation from earth-centered inertial (ECI) to in-plane inertial (IPI) coordinates.
$[A_{IPR}]$	Matrix transformation from ECI to in-plane rotating (IPR) coordinates.
A_{Ii}, A_{Ci}, A_{Ni}	Components of $\bar{A}_a$ along the input, cross, and normal axes of the *ith* accelerometer.
A'_{Ii}, A_{Oi}, A_{Si}	Components of $\bar{A}_a$ along the input, output, and spin axes of the *ith* gyroscope.
$[A_{LCI}]$	Matrix transformation from ECI to launch-centered inertial (LCI) coordinates.
$[A_{LCR}]$	Matrix transformation from ECI to launch-centered rotating (LCR) coordinates.
$[A_P]$	Matrix transformation from ECI to platform coordinates.
$[A_{Pm}]$	Matrix transformation from platform coordinates to misaligned platform coordinates.
$[A_{Ti}]$	Matrix transformation from earth-centered rotating (ECR) to the *ith* tracker coordinates.
$\bar{A}_a$	Nongravitational acceleration of the missile.
$[A_{ai}]$	Matrix transformation from platform coordinates to the *ith* accelerometer coordinates.
$[A_{gi}]$	Matrix transformation from platform coordinates to the *ith* gyroscope coordinates.
$\Delta\bar{A}_A$	Vector of the errors in the sensed accelerations of the three accelerometers.
$\Delta\bar{A}_{Dik}$	Vector of the acceleration error due to the *kth* drift rate component of the *ith* gyroscope.
$\Delta\bar{A}_M$	Vector of the acceleration errors due to platform misalignment.
$\Delta\bar{A}_a$	Total vector error in the sensed acceleration.
ΔA_i	Component of $\Delta\bar{A}_a$ for the *ith* accelerometer.
[B]	Matrix of the partial derivatives of the measured quantity with respect to the missile's coordinates relative to the tracker.
C_i	Coefficients in the expressions for the standard deviations of the external measurements, where i runs from 1 to 24.

G_{Ii}, G_{Ci}, G_{Ni}	Components of gravity along the input, cross, and normal axes of the *ith* accelerometer.
G'_{Ii}, G_{Oi}, G_{Si}	Components of gravity along the input, output, and spin axes of the *ith* gyroscope.
$[H]$	Parameter transformation matrix.
K_i	Error sensitivity coefficients associated with the IMU, where i runs from 1 to 45.
$[Q(t)]$	State covariance matrix.
R	Nominal range between the missile and a tracker.
R_B	Biased range between the missile and the actual tracker.
R_{BO}	True range between the missile and the actual tracker.
R_M	Measured value of R.
$\dot{R}_M$	Measured value of $\dot{R}$.
$\bar{R}_P$	Vector position of the missile in ECI coordinates as determined by the IMU.
$\bar{R}_m$	Nominal vector position of the missile in ECI coordinates.
R_{ref}	Reference range used in the hyperbolic range measurement.
δR	Random variation in the range measurement.
$\delta\dot{R}$	Random variation in the range rate measurement.
δR_B	Bias in the range measurement.
$\delta\dot{R}_B$	Bias in the range rate measurement.
δR_{B1}	Tropospheric bias in the range measurement.
$\delta\dot{R}_{B1}$	Tropospheric bias in the range rate measurement.
$d\bar{X}$	Best estimate of the state vector.
$d\bar{X}_C$	Actual value of the state vector.
x_R, y_R, z_R	Coordinates of the missile relative to the nominal tracker position.
x_{RB}, y_{RB}, z_{RB}	Coordinates of the missile relative to the actual tracker position.
Z	Generalized symbol for an external measurement.
dZ	Difference between the measurement Z and its nominal value.
$\alpha_i, \beta_i, \gamma_i$	Angular rotations specifying the orientation of the *ith* accelerometer relative to the platform axes.
$\alpha'_i, \beta'_i, \gamma'_i$	Angular rotations specifying the orientation of the *ith* gyroscope relative to the platform axes.

β	Nominal azimuth angle of the missile relative to the tracker.
β_B	Biased azimuth angle of the missile relative to the tracker.
β_M	Measured value of β.
β_0	Launch site longirude.
β_{Ti}	Longitude of the *ith* tracker.
$\delta\beta$	Random variation in the azimuth angle measurement.
$\delta\beta_B$	Bias in the measurement of β.
Γ	Greenwich Hour Angle.
Γ_0	Initial value of Γ.
γ	Nominal elevation angle of the missile relative to the tracker.
γ_B	Biased elevation angle of the missile relative to the actual tracker.
γ_{B0}	True elevation angle of the missile relative to the actual tracker.
γ_M	Measured value of γ.
$\delta\gamma$	Random variation in the measurement of the elevation angle.
$\delta\gamma_B$	Bias in the measurement of the elevation angle.
$\delta\gamma_{B1}$	Tropospheric bias in the measurement of the elevation angle.
$[\Lambda]$	Coefficient matrix in the differential equation for Φ.
λ_0	Latitude of the launch site.
λ_{Ti}	Latitude of the *ith* tracker.
μ	Earth's gravitational constant.
σ_R	Standard deviation of the range measurement.
$\sigma_{\dot{R}}$	Standard deviation of the range rate measurement.
σ_{HR}	Standard deviation of the hyperbolic range measurement.
σ_{HRRT}	Standard deviation of the hyperbolic range rate measurement.
σ_β	Standard deviation of the azimuth angle measurement.

σ_{γ}	Standard deviation of the elevation angle measurement.
$[\Phi]$	Upper left hand 6 × 6 matrix of Φ.
$\dot{\Phi}_{ki}$	Drift angular rate of the *ith* gyroscope due to the *kth* source of drift.
$[\Phi(t,t_0)]$	State transition matrix between time t_0 and time t.
ψ_0	Path azimuth angle of the missile at launch.
Ω_E	Spin angular rate of the earth.

I. INTRODUCTION

The RTRAJ program described in this report is designed to produce a best estimate of the performance of a moving vehicle, based on measurements made by an onboard inertial measuring unit (IMU) or external trackers, or both. The performance of a vehicle can be specified by a set of state variables which includes the vehicle position and velocity, and may include the error sensitivity coefficients of an onboard IMU. For the purposes of the analysis, it is convenient to define these state variables by their differential deviations from a nominal flight path and nominal IMU characteristics. It is the purpose of the program to make a best estimate of these differential variations, which constitute the state vector of the system. In addition, the precision of the estimate of the elements of the state vector is also determined, in the form of a covariance matrix whose diagonal elements are the variances of the state variables and whose off-diagonal terms are a measure of the covariance between state variables. In such a system, the position and velocity indicated by the IMU is, in the absence of any external measurements, the best and only estimate of the actual position and velocity of the vehicle. Under these conditions, the estimates of the deviations of the state variables from their nominal values are identically zero. However, the accelerations sensed by the IMU are functions of the state variables associated with the IMU, with the result that a priori standard deviations in these IMU parameters are propagated into standard deviations in position and velocity as time advances, with a corresponding modification of the position and velocity elements of the covariance matrix. At the same time, the standard deviations of the IMU parameters remain constant.

If, in addition to the IMU output, measurements are available from external trackers in the form of range, range rate, azimuth, or elevation angle determinations, these data can be used to improve the estimates of the state variables and their standard deviations. This updating of the estimates is accomplished by means of a sequential Kalman filter, which combines the current best estimate of the state variables with a given measured value, the standard deviation of the measurement, and the sensitivity of the state variables to this particular measurement to provide a new best estimate of the state variables. At the same time, these measurement data are also used to update the standard deviations of the estimates of the state variables. This procedure enables the user to estimate the deviation of the system from its nominal performance and to improve his confidence in this estimate. It should be noted that use of this Kalman filter assumes that the individual measurements are processed sequentially and that they are uncorrelated. In addition, if unmodeled errors are present, such as unknown biases in tracker positions or in the measurements, the resulting estimate of the vehicle performance will deviate from the actual performance. Although such biases could be modeled as additional state variables to be estimated, this option is not available in the present version of the program.

As an example of the type of problem to which the program is applicable, consider the flight of a ballistic missile with a period of powered flight followed by an interval of free fall to impact. It is assumed that vehicle position and velocity are computed by an onboard IMU and that this information is telemetered to ground observers. During powered flight, the motion of the missile is also observed by ground trackers which each measure the instantaneous range to the missile at specified measurement intervals. Initially, with the missile on the launch pad, there is no deviation in either position or velocity from that of the nominal flight path. However, the error sensitivity parameters of the particular IMU may deviate from their nominal values. Similarly, the a priori standard deviations of the position and velocity are initially zero, whereas those of the IMU parameters are not. When the missile is launched, it flies a path such that the accelerations sensed by the IMU are those specified for the nominal trajectory, but, since the IMU is in error, the actual missile deviates from the nominal trajectory. As indicated above, if the IMU were the only source of information, this deviation from the nominal flight path could not be detected. However, when the tracking information is incorporated into the estimation procedure, it is possible not only to obtain an estimate of the deviations in position and velocity from the nominal trajectory but also of the nonstandard values of the IMU parameters. In addition, an improved estimate of the standard deviations of these estimates of the state variables is achieved. If no further measurements are made with either the IMU or the external trackers, the state variables and the associated covariance matrix can be propagated from their values at cutoff along the trajectory to impact. The estimated and actual displacement of the impact point from that of the nominal trajectory can then be determined, as well as the dimensions and orientation of the error ellipse about that point.

Another example is tracking a ballistic missile during that portion of its flight from cutoff to impact. In this case, there is no IMU operating, but a set of ground trackers is measuring the range rate of the missile relative to each tracker. It is now necessary to specify a nominal initial position and velocity of the missile at cutoff, which determines the nominal flight path. In addition, the initial deviations of the actual missile position and velocity from these nominal conditions, as well as their a priori standard deviations, can be introduced. As in the previous case, the actual deviations from the nominal flight path are determined, as well as the best estimate of these deviations, based on combining the a priori estimates with the tracker data by means of the Kalman filter. In this way, the user obtains an estimate of the location of the actual impact as well as the confidence in this estimate.

A final example is the application to a navigational satellite system in which trackers are mounted on satellites whose ephemerides are known. As in the previous example, the user vehicle, which could be a ballistic missile, another satellite, an aircraft, or a terrestrial vehicle, is observed by the trackers and its estimated and actual deviation from its specified nominal path are determined, together with the precision of this estimate.

This report is intended as a user's manual for the operation of the RTRAJ program and not as a programmer's manual, and thus it does not contain the program listing or flow charts. The organization of the report is as follows: Section II presents the analysis necessary to describe the vehicle motion, the inertial measuring unit, the propagation of the state vector and the state covariance matrix, and, finally, the operation of the Kalman filter to update the performance estimate on the basis of the external measurements. Section III describes the various options available in the program. Section IV delineates the various types of tracking measurements available and how they are processed to provide input for the Kalman filter. Section V discusses the operation of the program, including both the configuration of the input deck and the various output formats which can be invoked. Finally, the appendix outlines the various coordinate systems used in the program and the transformation matrices relating them.

The program is written in FORTRAN IV and has been implemented on an IBM 370/158 computer.

II. ANALYSIS

VEHICLE MOTION

The nominal motion of a vehicle in the earth's gravitational field is defined by the equation

$$\ddot{\overline{R}}_m = -\frac{\mu \overline{R}_m}{R_m^3} + \overline{A}_a , \qquad (1)$$

where $\overline{R}_m$ is the vehicle position vector relative to the earth's center, μ is the earth's gravitational constant, and $\overline{A}_a$ is the nongravitational acceleration including thrust and aerodynamic forces.

The motion of the vehicle, as indicated by the IMU, is defined by the equation

$$\ddot{\overline{R}}_P = -\frac{\mu \overline{R}_P}{R_P^3} + \overline{A}_a + \Delta\overline{A}_a , \qquad (2)$$

where $\overline{R}_P$ is the vehicle position vector as indicated by the IMU, and $\Delta\overline{A}_a$ is the acceleration error resulting from uncertainties in the accelerometer parameters as well as platform misalignment and drift.

INERTIAL MEASURING UNIT ERROR ANALYSIS

The motion of the vehicle is indicated by an onboard inertial measuring unit, which consists of three accelerometers fixed to a stable platform with a 56-deg angle between the sensitive axes of each pair of accelerometers. The stabilization of the platform is accomplished by three single-degree-of-freedom gyroscopes mounted with mutually perpendicular input axes. This configuration is used unless other instrument orientation angles are specified in the input deck. It is assumed that the input acceleration is expressed in platform coordinates even though these coordinates may be misaligned relative to their idealized position. This input acceleration is represented as a column vector, $\overline{A}_a^{(P)}$.

Accelerometer Errors

It is convenient for discussion to project the applied acceleration into accelerometer axes by the following transformation:

$$\bar{A}_a^{(a_i)} = \begin{bmatrix} A_{Ii} \\ A_{Ci} \\ A_{Ni} \end{bmatrix} = [A_{ai}] \times \bar{A}_a^{(P)} , \tag{3}$$

where $\bar{A}_a^{(a_i)}$ is the acceleration in the instrument coordinates of the ith accelerometer and A_{Ii}, A_{Ci}, and A_{Ni} are the components along the input, cross, and normal axes, respectively, of the ith accelerometer.

There are six sources of error associated with each accelerometer; the error in the sensed acceleration for the ith accelerometer is

$$\Delta A_i = \sum_{k=3}^{8} K_{3(k-2)+i}\gamma_{ki} , \tag{4}$$

where the K's are the error sensitivity coefficients, and the γ's are given as

$$\gamma_{3i} = 1$$

$$\gamma_{4i} = A_{Ii} - G_{Ii}$$

$$\gamma_{5i} = A_{Ii}^2 - G_{Ii}^2$$

$$\gamma_{6i} = A_{Ii}^3 - G_{Ii}^3$$

$$\gamma_{7i} = A_{Ci} - G_{Ci}$$

$$\gamma_{8i} = A_{Ni} - G_{Ni}$$

where G_{Ii}, G_{Ci}, and G_{Ni} are the components of gravity along the I, C, and N axes, respectively, just prior to launch. The G's are only used when gravity compensation is used; otherwise, they are set to zero.

The errors in the sensed accelerations for the three accelerometers can be represented as a column vector of the form

$$\Delta\bar{A}_A^{(I)} = \begin{bmatrix} \Delta A_1 \\ \Delta A_2 \\ \Delta A_3 \end{bmatrix} , \tag{5}$$

which can be transformed to platform coordinates by the relation

$$\Delta\bar{A}_A^{(P)} = [R]^{-1}\bar{A}_A^{(I)} , \tag{6}$$

where R is the matrix transformation from platform coordinates to the three accelerometer input axes. Since these axes are not orthogonal, it is necessary to use the inverse of R to accomplish the transformation in Eq. (6).

Platform Misalignment Errors

If the inertial platform is initially misaligned by small angular rotations of K_1, K_2, and K_3 about the y, z, and x platform axes, respectively, the resulting acceleration errors introduced along the unperturbed platform axes are given by the relation

$$\Delta\bar{A}_M^{(P)} = \begin{bmatrix} 0 & K_2 & K_1 \\ K_2 & 0 & -K_3 \\ -K_1 & K_3 & 0 \end{bmatrix} \bar{A}_A^{(P)} . \tag{7}$$

Gyroscope Drift Errors

In addition to the initial platform misalignment, the drift of the three stabilizing gyroscopes may result in further misalignment as time goes on. To determine the drift rates of these gyroscopes, it is again convenient to project the applied acceleration along the instrument axes by the relation

$$\bar{A}_A^{(gi)} = \begin{bmatrix} A'_{Ii} \\ A_{Oi} \\ A_{Si} \end{bmatrix} = [A_{gi}]\ \bar{A}_a^{(P)} \ , \tag{8}$$

where A'_{Ii}, A_{Oi}, and A_{Si} are the acceleration components along the input, output, and spin axes, respectively, of the *ith* gyroscope.

There are eight sources of gyroscope drift; the drift rate of the *ith* gyroscope due to the *kth* source of drift is given by

$$\dot{\phi}_{ki} = K_{3(k-2)+i}\mu_{ki} \qquad (k = 1 \text{ through } 16) \ , \tag{9}$$

where, as in the case of the accelerometers, the K's are the error sensitivity coefficients, and the μ's are given by the following relations:

$$\mu_{9i} = 1$$

$$\mu_{10i} = A'_{Ii} - G'_{Ii}$$

$$\mu_{11i} = A_{Oi} - G_{Oi}$$

$$\mu_{12i} = A_{Si} - G_{Si}$$

$$\mu_{13i} = A'_{Ii}A_{Oi} - G'_{Ii}G_{Oi}$$

$$\mu_{14i} = A'_{Ii}A_{Si} - G'_{Ii}G_{Si}$$

$$\mu_{15i} = A_{Oi}A_{Si} - G_{Oi}G_{Si}$$

$$\mu_{16i} = T - T_0$$

where G'_{Ii}, G_{Oi}, and G_{Si} are the components of gravity along the input, output, and spin axes, respectively, immediately prior to launch. As in the case of the accelerometers, these G terms are omitted if gravity compensation is not used. The last term in Eq. (9) is a time-dependent drift rate and is proportional to the elapsed time since launch, $T - T_0$.

As a result of these drift rates, small angular rotations of the platform develop about the three mutually perpendicular gyroscope input axes and are given by

$$\phi_{ki} = \int_{T_0}^{T} \dot{\phi}_{ki}\, dt \; . \tag{10}$$

The resulting acceleration error in unperturbed platform coordinates due to the k*th* drift rate of the i*th* gyroscope is given by

$$\Delta\bar{A}_{D_{ik}}^{(P)} = [A_{gi}]^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\phi_{ik} \\ 0 & \phi_{ik} & 0 \end{bmatrix} \bar{A}_A^{(gi)} \; . \tag{11}$$

Combined Acceleration Errors

All of the acceleration errors discussed above make up the quantity $\Delta\bar{A}_a$ in Eq. (2). Since this equation is expressed in ECI coordinates, the value of the acceleration error term is given by

$$\Delta\bar{A}_a = [A_p]^T \left[\Delta\bar{A}_A^{(P)} + \Delta\bar{A}_M^{(P)} \sum_{L=1}^{3} \sum_{k=9}^{16} \Delta\bar{A}_{D_{ik}}^{(P)} \right] . \tag{12}$$

This expression involves a total of 45 error sensitivity parameters, represented by the K's which, for an ideal system, would all be zero with the exception of the three scale factor parameters, K_{4+i}, which should be unity.

THE STATE VECTOR

The state vector $d\bar{X}$ that describes this system has 51 components, including the small variations from nominal of the three missile position components, the three missile velocity components, and the 45 IMU parameters described above. Thus, given the nominal flight path and nominal IMU parameters, the system performance can be determined by solving for the state vector as a function of time, as $d\bar{X}(t)$.

THE STATE TRANSITION MATRIX (VEHICLE MODEL)

The state vector at time t is related to its value at time t_0 through the state transition matrix $\Phi(t,t_0)$ by the relation

$$d\bar{X}(t) = [\Phi(t,t_0)]\, d\bar{X}(t_0) \ . \tag{13}$$

The differential equation for $\Phi(t,t_0)$ is obtained by first differentiating Eq. (13) with respect to time to give

$$d\dot{\bar{X}}(t) = [\dot{\Phi}(t,t_0)]\, d\bar{X}(t_0) \ . \tag{14}$$

Elimination of $d\bar{X}(t_0)$ between Eq. (13) and (14) results in

$$d\dot{\bar{X}}(t) = [\dot{\Phi}(t,t_0)]\, [\Phi(t,t_0)]^{-1}\, d\bar{X}(t) \ . \tag{15}$$

However, $d\dot{\bar{X}}(t)$ can also be expressed as

$$d\dot{\bar{X}}(t) = \left[\frac{\partial\dot{\bar{X}}}{\partial\bar{X}}\right] d\bar{X}(t) \ . \tag{16}$$

Combination of Eqs. (15) and (16) gives

$$[\dot{\Phi}(t,t_0)] = [\Lambda]\, [\Phi(t,t_0)] \ , \tag{17}$$

which is the desired differential equation for $\Phi(t,t_0)$, and where

$$[\Lambda] = \left[\frac{\partial\dot{\bar{X}}}{\partial\bar{X}}\right] . \tag{18}$$

The matrix $[\Lambda]$ is a square matrix which can be partitioned as follows:

$$
\Lambda = \left[\begin{array}{c:c:c}
\dfrac{\partial \dot{\bar{R}}_p}{\partial \bar{R}_p} & \dfrac{\partial \dot{\bar{R}}_p}{\partial \dot{\bar{R}}_p} & \dfrac{\partial \dot{\bar{R}}_p}{\partial K_j} \quad (3 \times 45) \\
\hdashline
\dfrac{\partial \ddot{\bar{R}}_p}{\partial \bar{R}_p} & \dfrac{\partial \ddot{\bar{R}}_p}{\partial \dot{\bar{R}}_p} & \dfrac{\partial \ddot{\bar{R}}_p}{\partial K_j} \quad (3 \times 45) \\
\hdashline
\dfrac{\partial \dot{K}_i}{\partial \bar{R}_p} & \dfrac{\partial \dot{K}_i}{\partial \bar{R}_p} & \dfrac{\partial \dot{K}_i}{\partial K_j} \quad (45 \times 45) \\
(45 \times 3) & (45 \times 3) &
\end{array}\right] \tag{19}
$$

The partitioned elements can be evaluated as

$$\left[\frac{\partial \dot{\bar{R}}_p}{\partial \bar{R}_p}\right] = [0] \quad (3 \times 3)$$

$$\left[\frac{\partial \dot{\bar{R}}_p}{\partial \dot{\bar{R}}_p}\right] = [I] \quad (3 \times 3)$$

$$\left[\frac{\partial \ddot{\bar{R}}_p}{\partial \bar{R}_p}\right] = -\frac{\mu}{R_p^3} [I] + \frac{3\mu}{R_p^5} [\bar{R}_p \bar{R}_p^T] \quad (3 \times 3)$$

$$\left[\frac{\partial \ddot{\bar{R}}_p}{\partial \dot{\bar{R}}_p}\right] = [0] \quad (3 \times 3)$$

$$\left[\frac{\partial \dot{\bar{R}}_p}{\partial K_j}\right] = [0] \quad (3 \times 45)$$

$$\left[\frac{\partial \dot{K}_i}{\partial \bar{R}_P}\right] = [0] \; (45 \times 3)$$

$$\left[\frac{\partial \dot{K}_i}{\partial \dot{\bar{R}}_P}\right] = [0] \; (45 \times 3)$$

$$\left[\frac{\partial \dot{K}_i}{\partial K_j}\right] = [0] \; (45 \times 45)$$

The final (3 × 45) element, $\partial \ddot{\bar{R}}_P / \partial K_j$, is evaluated by differentiating Eq. (2) with respect to the 45 IMU error sensitivity parameters in the expression defining $\Delta \bar{A}_a$.

Since the lower 45 rows of the Λ matrix are identically zero, it follows from Eq. (17) that the corresponding 45 rows in the Φ matrix are also zero. Since the Φ matrix is initially an identity matrix when t equals t_0, its lower 45 rows remain unchanged with time because the differential values of the IMU parameters remain fixed. Thus, to determine the new value of the state transition matrix, it is only necessary to determine the elements in the upper six rows of Φ, which involves the integration of 306 first-order differential equations in the form of Eq. (17). In addition, there are 24 equations of the form of Eq. (9) which must be integrated to determine the drift angles associated with the platform, and, finally, six first-order equations resulting from Eq. (1) to give the three components of missile position and velocity. Thus, a total of 336 first-order differential equations must be integrated to determine the nominal missile trajectory and the state transition matrix. The state transition matrix can then be used in Eq. (13) to propagate the state vector.

COVARIANCE MATRIX (PROPAGATION THROUGH TIME)

The covariance matrix associated with the state vector is a symmetrical 51 × 51 matrix where the diagonal elements equal the squares of the standard deviations or variances of the state variables, and where the off-diagonal elements are the covariances indicating the correlation between variables. Initially, the first six diagonal elements representing the variances of the three position and the three velocity coordinates of the vehicle are set equal to zero, since there is no uncertainty in position or velocity prior to launch. The other 45 diagonal elements are set equal to the variances of the IMU parameters, and the off-diagonal covariance terms are all zero. As the flight progresses, the covariance matrix can be propagated by means of the state transition matrix by the relation

$$[Q(t_k)] = [\Phi(t_k,t_j)]\ [Q(t_j)]\ [\Phi(t_k,t_j)]^T \ , \tag{20}$$

which computes the covariance matrix $Q(t_k)$ at time t_k from its previous value $Q(t_j)$. Because of the form of the state transition matrix Φ, the propagated form of the covariance matrix $Q(t_k)$ can develop nonzero values in the elements in the first six rows or first six columns, or both. Thus, the variances of the IMU parameters result in uncertainties in the measured values of the missile's position and velocity. Although this process results in correlation between the IMU parameters and both position and velocity, there is no correlation between the various IMU parameters, as evidenced by the fact that the lower right hand 45 × 45 element of the covariance matrix remains unchanged--having all elements zero except those on the diagonal, which remain equal to the fixed variances of the IMU parameters.

MODELING THE EXTERNAL MEASUREMENT VARIABLES

The foregoing analysis has considered only the effect of the variations of the IMU parameters on the accuracy of the determination of the missile's position and velocity. If, however, external measurements of the missile's motion are made with known precision, during its flight, these measurements can be combined with those of the guidance system to provide an improved estimate, not only of the errors in missile position and velocity but also of the errors in the IMU parameters. It is first necessary, however, to determine the most likely value of the measurement errors based on the current best estimate of the state vector. This is given by the relation

$$d\bar{Z}_e = [H]\, d\bar{X} \,, \tag{21}$$

where $d\bar{Z}_e$ is the most likely measurement error, and [H] is the parameter transformation matrix with a dimension of 1 × 51 and is represented as

$$[H] = \left[\frac{\partial Z}{\partial \bar{X}}\right] = \left[\frac{\partial Z}{\partial \bar{R}_P}, \frac{\partial Z}{\partial \dot{\bar{R}}_P}, \frac{\partial Z}{\partial K}\right] \,. \tag{22}$$

Since the external measurement is independent of the IMU parameters, the last 45 elements of [H] are zero. The expressions for the remaining six elements of the H matrix for various types of measurements are derived in Sec. IV of this report.

ESTIMATION OF STATE VECTOR AND COVARIANCE MATRIX

By means of a Kalman filter, the external measurements can be combined with observations of the inertial guidance system to provide an improved estimate of the elements of the state vector as well as of the state covariance matrix. Initially, at time t_0, the state transition matrix is set equal to an identity matrix and the integration of the equations of motion proceeds to time t_1, when the first measurement is made. At this point, the integration has generated the state transition matrix $\Phi(t_1,t_0)$, which is now used to propagate the state vector and the state covariance to time t_1 by means of Eqs. (13) and (20). These propagated matrices are represented as $d\bar{X}(t_1/0)$ and $Q(t_1/0)$, where the zero in the argument means that no measurement data have yet been incorporated.

These matrices can now be updated by means of the Kalman filter relations as follows:[1]

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[1] R. H. Battin, *Astronautical Guidance*, McGraw-Hill Book Company, Inc., New York, 1964.

$$d\bar{X}(t_1/1) = d\bar{X}(t_1/0) + \frac{[Q(t_1/0)]\ [H_1]^T \left[d\bar{Z}_1 - [H_1]\ [d\bar{X}(t_1/0)]\right]}{[H_1]\ [Q(t_1/0)]\ [H_1]^T + \sigma_1^2} \tag{23}$$

$$[Q(t_1/1)] = [Q(t_1/0)] - \frac{[Q(t_1/0)]\ [H_1]^T\ [H_1]\ [Q(t_1/0)]}{[H_1]\ [Q(t_1/0)]\ [H_1]^T + \sigma_1^2} \tag{24}$$

where $d\bar{Z}_1$ is the difference between the actual measurement and that of the nominal reference trajectory, $[H_1]$ is the H matrix row corresponding to measurement number one, and σ_1 is the standard deviation of measurement number one.

If a total of n measurements are made at time t_1, Eqs. (23) and (24) are applied n times, after which the best estimates of the state vector and the state covariance are $d\bar{X}(t_1/n)$ and $Q(t_1/n)$. After all of the measurements have been processed, the state transition matrix $\Phi(t_1,t_1)$ is set equal to an identity matrix and the above procedure is repeated until missile impact occurs. At this time, the first three elements of the state vector, representing the deviations in the estimated missile position from the nominal trajectory impact point, are used to determine the displacement of the estimated impact point. In addition, the state covariance matrix can be used to determine the major and minor axes, as well as the orientation, of the error ellipse on the ground about the estimated impact point.

III. PROGRAM OPTIONS

Since the computer program described in this report can be applied to a wide variety of problems, there are a number of options which can and must be specified in any given run. These options are described below; the mechanics of invoking them are shown in Sec. V.

MODE OF OPERATION

In Sec. II, it is shown that the nominal performance of a missile is specified by the solution of Eq. (1), which assumes no errors in the IMU parameters. The nonnominal performance of this same missile can be characterized by a state vector consisting of the position and velocity errors relative to the nominal trajectory, as well as the 45 nonzero values of the IMU parameters. Before discussing the modes of operation, it is convenient to consider two forms of this state vector. The first, $d\overline{X}_c$, describes the actual performance of the missile, since its elements are the actual deviations in position and velocity from the nominal trajectory and the actual values of the errors in the IMU parameters. These elements of $d\overline{X}_c$ can be selected from gaussian distributions with appropriately specified standard deviations. In any event, these values are not available to the onboard guidance system. The second state vector, $d\overline{X}$, represents the performance of the missile as estimated by the onboard guidance, with or without external measurements.

Mode 0 (Propagation of Covariance)

In this case, it is assumed that the missile flies the nominal trajectory; the elements of $d\overline{X}_c$ are thus identically zero. Initially, all of the elements of the state vector $d\overline{X}$ are also zero, since at this time there is no reason to doubt the IMU output. In the covariance matrix, the only nonzero values are the diagonal elements representing the variances of the IMU parameters. As the missile progresses along its trajectory, in the absence of any external measurement, the values of $d\overline{X}$ and $d\overline{X}_c$ remain zero, and the state covariance matrix is propagated in accordance with Eq. (20). If external measurements are made, the values of $d\bar{X}$ may still remain zero unless there are biases in the measurements. In the presence of a measurement bias, the term $d\bar{Z}_1$ in Eq. (23) is no longer zero and the resulting values of $d\bar{X}$ are likewise nonzero. At the same time, the covariance matrix is modified by the measurement process of Eq. (24) to give an improved estimate of the variances of the state variables as well as the off-diagonal covariances. It should be emphasized that the resulting covariance matrix is independent of the biases in the measurement and that the nonzero values of $d\bar{X}$ represent an erroneous estimate of the missile performance, since the missile actually is on the nominal trajectory ($d\bar{X}_c$ is zero).

This mode of operation can also be used to estimate the nonnominal performance of the missile as characterized by the state vector $d\bar{X}_c$. To do this, the initial position and velocity errors relative to the nominal trajectory, as well as the IMU parameters, are selected from gaussian distributions to form the state vector $d\bar{X}_c$, where the state vector $d\bar{X}$ is initially zero. Both $d\bar{X}_c$ and $d\bar{X}$ are propagated by means of Eq. (13) up to the time of the first external measurements, when $d\bar{X}$ is still zero. The value of $d\bar{X}$ is then updated by incorporating the external measurements by means of Eq. (23), after which $d\bar{X}$ is again propagated to the time of the next measurements. As this procedure of propagating and updating proceeds, the estimate of the state vector $d\bar{X}$ should approach the actual state vector $d\bar{X}_c$, thus giving an improved estimate of the missile's performance. The alternate propagation and updating of the covariance matrix by means of Eqs. (20) and (24) gives the same result as that obtained above, since this propagation and updating is independent of the actual state vector $d\bar{X}_c$.

<u>Mode 1 (Generation of Data)</u>

This mode of operation is designed to produce a set of tracking data that can be used later as input to the Mode 2 operation of the program, in which an estimate of the missile performance is determined. The data so generated are of the same sort that might be provided by tracking stations during an actual missile firing.

To generate the appropriate tracking data, the missile flies along the nominal trajectory specified by Eq. (1). Each of the external tracking stations is assumed to be capable of measuring one or more of the following quantities: the range to the missile, its range rate, its azimuth angle, or its elevation angle. At specified time , for each of the tracking stations, the true values of these observables are determined. These values are then modified by introducing biases in the tracker position and in the measurement, and, finally, a random measurement error selected from a gaussian distribution whose standard deviation is that specified for the particular measurement process. These modified values are the desired observational data and are recorded totether with the tracker identification and the time of the measurement.

Mode 2 (Simulation)

In this mode of operation, the objective is to estimate the missile state vector $d\overline{X}$ by combining the IMU measurements with the external tracking data. As indicated above, these external tracking data can be provided either by the Mode 1 operation or by recording the tracking data from an actual missile flight. Initially, the elements of $d\overline{X}$, the estimated state vector, are set equal to zero since, in the absence of any external measurement, there is no reason to assume that the vehicle has deviated from the nominal trajectory. However, at each measurement time, the external measurements are used to update the estimated state vector $d\overline{X}$. These values are then used to determine the new values of the state variables by the relation

$$\overline{X}(t/n) = \overline{X}(t/m) + d\overline{X}(t/n) \quad , \tag{25}$$

where the arguments of $\overline{X}$ have the same meaning as those of $d\overline{X}$ in Eq. (23), and the m and n are the number of measurements before and after updating, respectively. Equation (25) results in a specification of a new nominal trajectory which should approach the true trajectory as the number of external measurements increases. It should be noted that this type of simulation, using Mode 1 to generate the data, is essentially equivalent to the Mode 0 operation, with a nonzero value of $d\overline{X}_c$.

Thus, the principal advantage of Mode 2 is its ability to utilize externally generated real tracking data. Otherwise, Mode 0 is preferable and uses less computer time per run.

FLIGHT PATH CHARACTERISTICS

Staging

For computational convenience, the flight path is divided into several time intervals or stages. At the beginning of each stage, data are input to describe the vehicle acceleration, the characteristics of the external tracking system, and the format and frequency of the printout. Thus, a given run might have a stage of powered flight in which the vehicle acceleration is specified and during which no tracking measurements are made. This could be followed by a stage of unpowered flight in which the missile is in free fall and during which external measurements are made by the ground trackers. Finally, this would be followed by another stage of free fall, but with no external measurements, continuing on to impact.

Acceleration Profile

The acceleration of the missile as a function of time can be input in tabular form by expressing the three components of acceleration at specified times. The acceleration at any other time is determined by an interpolation procedure in the program. The components of acceleration can be expressed in either a launch-centered inertial or launch-centered rotating coordinate system, depending on which system the inertial platform uses.

When the missile is in free fall under the influence of gravity, acceleration is determined analytically, with both drag and thrust set equal to zero.

Earth Model

The earth can be chosen to be either rotating or nonrotating and either oblate or spherical. If an oblate earth is chosen, the J_2 term is included in the potential function and the difference between geodetic and geocentric latitude is taken into account.

TRACKER CHARACTERISTICS

Number of Trackers

In its present form, the program can handle a maximum of 24 trackers, each of which can make a maximum of four different types of measurement at a given observation time.

Tracker Location

Each of the 24 trackers can be located on the ground, on an aircraft, or on a satellite. For a ground-based tracker, it is necessary to specify its latitude, longitude, and altitude relative to the rotating earth. It is assumed that the aircraft carrying trackers are flying great circle routes relative to the rotating earth. Thus, it is necessary to specify not only their latitude, longitude, and altitude, but also their velocity and heading as well as the time at which these conditions exist. From these initial position data, the position and velocity at some future time can be determined. The satellites bearing trackers are on either circular or elliptical orbits fixed in inertial space. As in the case of aircraft, it is necessary to specify their latitude, longitude, and altitude relative to the rotating earth at a given time, as well as the velocity, path elevation angle, and azimuth angle relative to inertial space. From these initial data, the position and velocity of the satellite at any other time can be determined.

A fourth type of tracker which can be introduced is an altimeter located in the missile. The most convenient way to implement this in the program is as a hypothetical shadow tracker that moves along the surface of the earth below the missile and measures the range to the missile. These range measurements are identical with the altitude measurements that would be produced by an altimeter in the missile. It should be noted that these altitude measurements are assumed to be relative to an ellipsoidal earth, since the program does not model local altitude variations of the earth's surface.

It is also possible to introduce biases in the latitude, longitude, altitude, and velocity of the initial positions of the trackers--with the exception of the shadow trackers. These biases can be specified directly or can be selected from gaussian distributions whose standard deviations have been specified.

Tracker Limitations

There are a number of restrictions to the operation of the trackers which can be set individually for each of the 24 trackers. The first of these is the time interval during which the tracker is in operation and available to make measurements. In the program, it is possible to specify a maximum of four time intervals during which a given tracker is operational.

It is also possible to set the minimum interval between observations for each tracker. It should be noted that, if a given tracker is capable of more than one type of measurement, all of these measurements are assumed to be made simultaneously and the interval specified above represents the minimum time before another set of such measurements can be made by the same tracker.

Finally, the ability of a tracker to observe a target may be restricted by the target elevation angle relative to the tracker. This can be specified in the form of a minimum and maximum permissible elevation angle for each tracker.

MEASUREMENT CHARACTERISTICS

Types of Measurement

There is a maximum of six different types of measurements which can be made by any one tracker: range, range rate, azimuth angle, elevation angle, hyperbolic range, and hyperbolic rate. The details of the various types of measurement are described in Sec. IV. Although six types are available, it is not anticipated that more than four would be used by any one tracker so that, with 24 trackers, the maximum number of measurements at any one time would be 96.

Measurement Biases

A bias can be specified for each type of measurement and each tracker that need not be the same for all trackers. The magnitudes of these biases can be input directly or selected from gaussian distributions with appropriate standard deviations.

Standard Deviations

For each type of measurement which can be made by a given tracker, it is necessary to assign a nonzero standard deviation that can vary from tracker to tracker.

IV. MEASUREMENT PROCEDURE

This section describes the mechanics of the measurement procedure at each measurement time.

TRACKER SELECTION

Each of the trackers is examined in sequence to determine whether it is available to make an observation at a given measurement time. If this condition is satisfied, the position and velocity of the nominal missile relative to this tracker are determined in tracker coordinates, represented by x_R, y_R, z_R, $\dot{x}_R$, $\dot{y}_R$, and $\dot{z}_R$. From these coordinates, the missile's range, range rate, azimuth angle, and elevation angle relative to the tracker are determined from the following relations:

Range $$R = \sqrt{x_R^2 + y_R^2 + z_R^2} \quad (26)$$

Range rate $$\dot{R} = \frac{x_R\dot{x}_R + y_R\dot{y}_R + z_R\dot{z}_R}{R} \quad (27)$$

Elevation angle $$\gamma = \tan^{-1}\left(\frac{z_R}{\sqrt{x_R^2 + y_R^2}}\right) \quad (28)$$

Azimuth angle $$\beta = \tan^{-1}\left(\frac{x_R}{y_R}\right) \quad (29)$$

At this point, a check is made to be sure that the elevation angle γ lies between the maximum and minimum values for the tracker. In addition, if the tracker is on a satellite, a check is made to determine whether the line of sight between the tracker and missile is unobstructed. If either of these conditions is violated, the tracker is not available and a new one, if available, must be selected.

After the above visibility tests have been satisfied, the biased values of the variables determined in Eqs. (26) through (29) are found by computing the position and velocity of the nominal missile relative to the biased tracker position in tracker coordinates represented by x_{RB}, y_{RB}, z_{RB}, $\dot{x}_{RB}$, $\dot{y}_{RB}$, and $\dot{z}_{RB}$. The biased versions of Eqs. (26) through (29) have the form

Biased range

$$R_B = R_{BO} + \delta R_B + \frac{\delta R_{B1}}{0.1 + 0.9 \sin \gamma_{BO}} \tag{30}$$

Biased range rate

$$\dot{R}_B = \frac{x_{RB}\dot{x}_{RB} + y_{RB}\dot{y}_{RB} + z_{RB}\dot{z}_{RB}}{R_{BO}} + \delta \dot{R}_B + \frac{\delta \dot{R}_{B1}}{0.1 + 0.9 \sin \gamma_{BO}} \tag{31}$$

Biased elevation angle

$$\gamma_B = \gamma_{BO} + \delta\gamma_B + \frac{\delta\gamma_{B1}}{0.1 + 0.9 \sin \gamma_{BO}} \tag{32}$$

Biased azimuth angle

$$\beta_B = \tan^{-1}\left(\frac{x_{RB}}{y_{RB}}\right) + \delta\beta_B \tag{33}$$

where

$$R_{BO} = \sqrt{x_{RB}^2 + y_{RB}^2 + z_{RB}^2}$$

and

$$\gamma_{BO} = \tan^{-1}\left(\frac{z_{RB}}{\sqrt{x_{RB}^2 + y_{RB}^2}}\right) .$$

The quantities δR_B, $\delta \dot{R}_B$, $\delta\beta_B$, and $\delta\gamma_B$ are the fixed measurement biases for the given tracker, whereas the terms involving δR_{B1}, $\delta \dot{R}_{B1}$, and $\delta\gamma_{B1}$

represent the measurement biases due to tropospheric effects that are elevation angle dependent.

MEASUREMENT TYPES

The computations indicated thus far in this section are made for each available tracker, regardless of the type of measurements it is capable of making. In the succeeding paragraphs describing the various measurement procedures, the computations are made only if the tracker is capable of the specified type of measurement.

Range Measurement

If the tracker is intended to measure range, the variance of the measurement is determined from the relation

$$\sigma_R^2 = C_1^2 + \frac{C_7}{\sin^2 \gamma} + C_{13} R^{C_{19}} , \tag{34}$$

where the first term is due to random time delays in the measuring system, the second to tropospheric refraction, and the third to the signal-to-noise ratio. Since the model used to compute the tropospheric term assumes that the tracker is on the ground looking up at the missile, it is probably desirable to omit this term when dealing with satellite-borne trackers by setting C_7 equal to zero.

The reference range R_C between the nominal tracker and missile positions is now set equal to the value of R computed in Eq. (26), and the measured range R_M is given by

$$R_M = R_B + \delta R , \tag{35}$$

where R_B is obtained from Eq. (30) and δR is a random variation in range selected from a gaussian distribution with a standard deviation σ_R given by Eq. (34).

The H matrix row associated with this range measurement is computed by first finding the B matrix row, which is given as

$$[B] = \left[\frac{\partial R}{\partial x_R} \; \frac{\partial R}{\partial y_R} \; \frac{\partial R}{\partial z_R} \; \frac{\partial R}{\partial \dot{x}_R} \; \frac{\partial R}{\partial \dot{y}_R} \; \frac{\partial R}{\partial \dot{z}_R}\right] \tag{36}$$

whose elements can be evaluated by means of Eq. (26) as

$$\begin{aligned}
\frac{\partial R}{\partial x_R} &= \frac{x_R}{R} \\
\frac{\partial R}{\partial y_R} &= \frac{y_R}{R} \\
\frac{\partial R}{\partial z_R} &= \frac{z_R}{R} \\
\frac{\partial R}{\partial \dot{x}_R} &= 0 \\
\frac{\partial R}{\partial \dot{y}_R} &= 0 \\
\frac{\partial R}{\partial \dot{z}_R} &= 0 \; .
\end{aligned} \tag{37}$$

The H matrix row is then determined by the relation

$$[H] = [B] \times [A] \; , \tag{38}$$

where the A matrix, which is derived in the appendix, has the form

$$[A] = \begin{bmatrix} \frac{\partial x_R}{\partial X_1} & \frac{\partial x_R}{\partial X_2} & \frac{\partial x_R}{\partial X_3} & \frac{\partial x_R}{\partial X_4} & \frac{\partial x_R}{\partial X_5} & \frac{\partial x_R}{\partial X_6} \\ \frac{\partial y_R}{\partial X_1} & \frac{\partial y_R}{\partial X_2} & \frac{\partial y_R}{\partial X_3} & \frac{\partial y_R}{\partial X_4} & \frac{\partial y_R}{\partial X_5} & \frac{\partial y_R}{\partial X_6} \\ \frac{\partial z_R}{\partial X_1} & \frac{\partial z_R}{\partial X_2} & \frac{\partial z_R}{\partial X_3} & \frac{\partial z_R}{\partial X_4} & \frac{\partial z_R}{\partial X_5} & \frac{\partial z_R}{\partial X_6} \\ \frac{\partial \dot{x}_R}{\partial X_1} & \frac{\partial \dot{x}_R}{\partial X_2} & \frac{\partial \dot{x}_R}{\partial X_3} & \frac{\partial \dot{x}_R}{\partial X_4} & \frac{\partial \dot{x}_R}{\partial X_5} & \frac{\partial \dot{x}_R}{\partial X_6} \\ \frac{\partial \dot{y}_R}{\partial X_1} & \frac{\partial \dot{y}_R}{\partial X_2} & \frac{\partial \dot{y}_R}{\partial X_3} & \frac{\partial \dot{y}_R}{\partial X_4} & \frac{\partial \dot{y}_R}{\partial X_5} & \frac{\partial \dot{y}_R}{\partial X_6} \\ \frac{\partial \dot{z}_R}{\partial X_1} & \frac{\partial \dot{z}_R}{\partial X_2} & \frac{\partial \dot{z}_R}{\partial X_3} & \frac{\partial \dot{z}_R}{\partial X_4} & \frac{\partial \dot{z}_R}{\partial X_5} & \frac{\partial \dot{z}_R}{\partial X_6} \end{bmatrix} . \qquad (39)$$

Thus, Eq. (38) gives the H matrix in the form shown in Eq. (22), where Z is replaced by R as the measured quantity.

Range Rate Measurement

If the tracker is intended to measure range rate, the variance of the measurement is determined from the relation

$$\sigma_{\dot{R}}^2 = C_2^2 + \frac{C_8}{\sin^2 \gamma} + C_{14} R^{C_{20}} \qquad (40)$$

in a form similar to that used for the range variance in Eq. (34).

The reference range rate $\dot{R}_C$ between the nominal tracker and missile positions is set equal to $\dot{R}$, computed in Eq. (27), and the measured range rate $\dot{R}_M$ is given by

$$\dot{R}_M = \dot{R}_B + \delta\dot{R} , \tag{41}$$

where $\dot{R}_B$ is obtained from Eq. (31) and $\delta\dot{R}$ is a random variation in range rate which can be selected from a gaussian distribution with a standard deviation given by Eq. (40).

As in the case of range measurement, another row of the B matrix is determined in the form

$$[B] = \left[\frac{\partial\dot{R}}{\partial x_R} \ \frac{\partial\dot{R}}{\partial y_R} \ \frac{\partial\dot{R}}{\partial z_R} \ \frac{\partial\dot{R}}{\partial\dot{x}_R} \ \frac{\partial\dot{R}}{\partial\dot{y}_R} \ \frac{\partial\dot{R}}{\partial\dot{z}_R}\right] , \tag{42}$$

where the elements of this row matrix can be determined from Eq. (27) in the form

$$\begin{aligned}
\frac{\partial\dot{R}}{\partial x_R} &= \frac{\dot{x}_R}{R} - \frac{\dot{R}x_R}{R^2} \\
\frac{\partial\dot{R}}{\partial y_R} &= \frac{\dot{y}_R}{R} - \frac{\dot{R}y_R}{R^2} \\
\frac{\partial\dot{R}}{\partial z_R} &= \frac{\dot{z}_R}{R} - \frac{\dot{R}z_R}{R^2} \\
\frac{\partial\dot{R}}{\partial\dot{x}_R} &= \frac{x_R}{R} \\
\frac{\partial\dot{R}}{\partial\dot{y}_R} &= \frac{y_R}{R} \\
\frac{\partial\dot{R}}{\partial\dot{z}_R} &= \frac{z_R}{R} .
\end{aligned} \tag{43}$$

The corresponding row of the H matrix is again obtained by means of Eq. (38).

Elevation Angle Measurement

If the tracker is intended to measure elevation angle, the variance of the measurement is determined from the relation

$$\sigma_\gamma^2 = C_3^2 + C_9 \frac{\cos\gamma}{\sin^2\gamma} + C_{15}R^{C_{21}} , \tag{44}$$

where the tropospheric term has a somewhat different dependence on the elevation angle than in the case of the range and range rate measurements.

The reference elevation angle γ_C of the nominal missile relative to the nominal tracker is set equal to the value of γ computed in Eq. (28). The measured elevation angle γ_M is given by

$$\gamma_M = \gamma_B + \delta\gamma , \tag{45}$$

where γ_B is given by Eq. (32) and $\delta\gamma$ is a random elevation angle variation selected from a gaussian distribution with a standard deviation σ_γ given by Eq. (44).

The row of the B matrix corresponding to this measurement has the form

$$[B] = \left[\frac{\partial\gamma}{\partial x_R} \; \frac{\partial\gamma}{\partial y_R} \; \frac{\partial\gamma}{\partial z_R} \; \frac{\partial\gamma}{\partial \dot{x}_R} \; \frac{\partial\gamma}{\partial \dot{y}_R} \; \frac{\partial\gamma}{\partial \dot{z}_R}\right] , \tag{46}$$

where the elements of this row matrix can be determined from Eq. (28) in the form

$$\frac{\partial\gamma}{\partial x_R} = -\frac{x_R z_R}{R^2\sqrt{x_R^2 + y_R^2}}$$

$$\frac{\partial\gamma}{\partial y_R} = -\frac{y_R z_R}{R^2\sqrt{x_R^2 + y_R^2}}$$

$$\frac{\partial\gamma}{\partial z_R} = -\frac{\sqrt{x_R^2 + y_R^2}}{R^2} \tag{47}$$

$$\frac{\partial\gamma}{\partial \dot{x}_R} = 0$$

$$\frac{\partial\gamma}{\partial \dot{y}_R} = 0$$

$$\frac{\partial\gamma}{\partial \dot{z}_R} = 0 \ .$$

The corresponding row of the H matrix is again obtained by means of Eq. (38).

Azimuth Angle Measurement

If the tracker is intended to measure azimuth angle, the variance of this measurement is given by

$$\sigma_\beta^2 = C_4^2 + C_{16}R^{C_{22}} \ , \tag{48}$$

where it is seen that no tropospheric term is present.

The reference azimuth angle β_C of the nominal missile relative to the nominal tracker is set equal to the value of β obtained from Eq. (29); the measured azimuth angle β_M is given by

$$\beta_M = \beta_B + \delta\beta \ , \tag{49}$$

where β_B is given by Eq. (33) and $\delta\beta$ is a random azimuth angle variation which can be selected from a gaussian distribution having a standard deviation σ_β given by Eq. (48).

The row of the B matrix corresponding to this measurement has the form

$$[B] = \left[\frac{\partial\beta}{\partial x_R} \ \frac{\partial\beta}{\partial y_R} \ \frac{\partial\beta}{\partial z_R} \ \frac{\partial\beta}{\partial \dot{x}_R} \ \frac{\partial\beta}{\partial \dot{y}_R} \ \frac{\partial\beta}{\partial \dot{z}_R}\right] \ , \tag{50}$$

where the elements of this row can be determined from Eq. (29) as follows:

$$\begin{aligned}
\frac{\partial\beta}{\partial x_R} &= -\frac{x_R}{x_R^2 + y_R^2} \\
\frac{\partial\beta}{\partial y_R} &= -\frac{x_R}{x_R^2 + y_R^2} \\
\frac{\partial\beta}{\partial z_R} &= 0 \\
\frac{\partial\beta}{\partial \dot{x}_R} &= 0 \\
\frac{\partial\beta}{\partial \dot{y}_R} &= 0 \\
\frac{\partial\beta}{\partial \dot{z}_R} &= 0 \ .
\end{aligned} \tag{51}$$

The corresponding row of the H matrix is determined by means of Eq. (38).

Hyperbolic Range Measurement

This measuring system involves a one-way range measurement in which each tracker radiates a continuous signal with a modulation that identifies its time of origin when received by the missile. At a given measurement time, the missile can determine the time of propagation from each of the observed trackers from the difference between the measurement time as measured by the onboard clock and the respective times of origin as determined by a ground-based clock. If both of these clocks are synchronized, the resulting propagation times can be used to determine the instantaneous ranges to each of the observing trackers. Since there is usually a drift between the two clocks, it is preferable to select one of the trackers as a reference and determine the differences in propagation time between each of the other trackers and the selected reference tracker. In this way, the error in the onboard clock can be eliminated.

The implementation of this method is initially identical with that described for the two-way range measurement. The standard deviation of the measurement is given by

$$\sigma_{HR}^2 = C_5^2 + \frac{C_{11}}{\sin^2 \gamma} + C_{17} R^{C_{23}} , \tag{52}$$

which is of the same form as Eq. (34) but differs in the values of C's because of the one-way nature of the measurement. The reference range R_C, the measured range R_M, the elements of the B matrix, and the corresponding elements of the H matrix are determined for each tracker as before.

To use the hyperbolic range method, at least two trackers must be observed at a given measurement time. If this condition is satisfied, the tracker with the minimum range is selected as a reference. A range difference measurement can then be determined for each of the other trackers from the difference of its measured range and that of the reference tracker. Similarly, the nominal range difference for each tracker is its nominal range minus that of the reference tracker. Finally, the elements of the H matrix for these range difference measurements are obtained from the difference of the corresponding elements of the H matrix computed above for the given tracker and those for the reference tracker in the form

$$\begin{aligned} H_i(\Delta R) &= \frac{\partial(\Delta R)}{X_i} \\ &= \left(\frac{\partial R}{\partial X_i}\right) - \left(\frac{\partial R_{ref}}{\partial X_i}\right) \qquad (53) \\ &= H_i(R) - H_i(R_{ref}) \ . \end{aligned}$$

Thus, the number of independent hyperbolic range measurements is one less than the number of observing trackers.

Another version of the hyperbolic range method eliminates the reduction in the number of measurements at each observation time after the first by saving the minimum range measurement and the nominal range from the previous observation time, as well as the corresponding row of the H matrix, $H_i(R_{min})$. This H matrix row is propagated forward to the new observation time by the relation

$$H(R_{ref}) = H(R_{min}) \ [\phi]^{-1} \ , \qquad (54)$$

where $[\phi]$ is the upper left hand 6 × 6 of the state transition matrix Φ. The saved values of the range measurement, the nominal range, and the propagated H matrix elements can now be used as the reference values to be subtracted out at the new observing time. Since these values can also be subtracted from the observations of the tracker which provided the previous minimum range, the number of independent hyperbolic range measurements is now equal to the number of observing trackers.

A final version of the hyperbolic range measurement method uses a different reference for each tracker--the previous measurement made by the given tracker. All of the range measurements and nominal ranges from the previous observation time must thus be saved, as well as the entire H matrix, which is then propagated forward to the present time by means of Eq. (54). The resulting matrix, together with the saved measured ranges and nominal ranges, can be subtracted from the corresponding data for the respective trackers to provide the desired hyperbolic range data. It is seen that, in this case, a hyperbolic range measurement can only be made by a tracker if it was observed at the previous observation time as well as at present. This approach has the advantage that it tends to cancel out the individual tracker position biases as well as the clock error; it is referred to as the delta range method.

Hyperbolic Range Rate Measurement

This measuring system is entirely analogous to the hyperbolic range method described above; it has a standard deviation of the form

$$\sigma_{HRRT} = C_6^2 + \frac{C_{12}}{\sin^2 \gamma} + C_{18} R^{C_{24}} \ . \tag{55}$$

In this case, the range rate for each tracker is determined onboard the missile by comparing the received frequency with the frequency of an onboard oscillator which presumably is matched to the radiated frequency. To eliminate the frequency error between the onboard oscillator and the radiated frequency, a differencing technique is used in which the data corresponding to the minimum range tracker are subtracted from the corresponding data for the other trackers to give the desired hyperbolic range rate measurements in a form appropriate for processing by the Kalman filter. This method also has the option of establishing a reference set of data by propagating the data from the minimum range tracker at the previous measuring time to the present time by means of the state transition matrix, in accordance with

Eq. (54). Although it could be done, the program does not provide for saving individual measurements and H matrix elements for all trackers as references for the next measurements by these systems, as was done in the delta range case.

V. RTRAJ PROGRAM OPERATION

INPUT DECK

With the exception of one card, the RTRAJ input deck uses the FORTRAN namelist method of data input. This subsection lists the input variables with their definitions and their default values in the event they are not specified. The namelists and the one formatted card input appear in their required order, although the order of the variables within a given namelist is immaterial.

Namelist/BIASIN/

IRUN Number of missile flights submitted in this input deck; a default value of one.

SDRANG Standard deviation in range, in feet, of the gaussian distribution from which the range measurement biases of the various trackers can be selected.

SDRRT Standard deviation in range rate, in feet per second, of the gaussian distribution from which the range rate measurement biases of the various trackers can be selected.

SDELEV Standard deviation in elevation angle, in milliradians, of the gaussian distribution from which the elevation angle measurement biases of the various trackers can be selected.

SDAZIM Standard deviation in azimuth angle, in milliradians, of the gaussian distribution from which the azimuth angle measurement biases of the various trackers can be selected.

SDRNG Standard deviation in range, in feet, of the gaussian distribution from which the tropospheric range measurement bias at 90-deg elevation angle for the various trackers can be selected.

SDRAT Standard deviation in range rate, in feet per second, of the gaussian distribution from which the tropospheric range rate measurement bias at 90-deg elevation angle for the various trackers can be selected.

SDELV — Standard deviation in elevation angle, in milliradians, of the gaussian distribution from which the tropospheric elevation angle measurement bias at 90-deg elevation angle for the various trackers can be selected.

SDLAT — Standard deviation in latitude, in feet, of the gaussian distribution from which the latitude biases of the various trackers can be selected.

SDLON — Standard deviation in longitude, in feet, of the gaussian distribution from which the longitude biases of the various trackers can be selected.

SDALT — Standard deviation in altitude, in feet, of the gaussian distribution from which the altitude biases of the various trackers can be selected.

SDVEL — Standard deviation in tracker velocity, in feet per second, of the gaussian distribution from which the biases in velocity of the various trackers can be selected.

SX(I) — The six standard deviations in missile position, in feet (I = 1,2,3), and velocity, in feet per second (I = 4,5,6), of the gaussian distributions from which missile position and velocity biases can be selected.

The default values of all these standard deviations are zero.

Formatted Card Input

The format for this lone input card is D10.0, I5, I5, and the variables are as follows:

PRNTDT — Interval in flight time, in seconds, at which output data are to be printed.

ISTG — A flag to indicate whether this stage is the last one of this flight.

0 This is the final stage of this flight.

1 Another stage follows.

<u>IBST</u> A flag to indicate the type of acceleration profile used; it can have an integral value from 0 to 2 with the following meanings:

- 0 The missile is in free fall and its acceleration is determined analytically.
- 1 The missile acceleration is input in tabular form as a function of time in launch-centered inertial coordinates. It also implies that the inertial platform associated with the IMU is stabilized in this same coordinate system.
- 2 The missile acceleration is input in tabular form as a function of time in launch-centered rotating coordinates. It also implies that the inertial platform is stabilized in this same coordinate system.

<u>Namelist/STATIN/</u>

<u>INPQ</u> A flag to describe the form of the input of the initial covariance matrix Q; it can have integral values between -4 and +4. If the sign of INPQ is negative, only the diagonal elements of the covariance matrix are entered, as DATQ; a positive sign means that the entire covariance matrix is entered, as Q. The magnitude of INPQ indicates the coordinate system in which the covariance matrix is expressed.

- 0,1 Earth-centered inertial.
- 2 In-plane inertial.
- 3 Earth-centered rotating.
- 4 In-plane rotating.

INPQ should only be specified in the first stage of the flight and then only if it differs from the default value of zero.

<u>INPX</u> A flag to determine the coordinate system in which the estimated state vector DX is expressed. It can have an integral value from 0 to 4 with the same meanings as

those for the magnitude of INPQ. As above, it is only necessary to specify INPX in the first stage of the flight and then only if it differs from the default value of zero.

IX — This flag controls the printout of the state variables X and the state vector DX and can have the integral values of 0 or 1 with the meanings:

- 0 Do not print X and DX.
- 1 Print X and DX.

The default value of IX is zero.

IQ — A flag to control the printout of the covariance matrix as well as the associated eigen values and eigen vectors. It can have integral values from 0 to 7 with the following meanings:

- 0,2,4,6 Do not print the upper left 6 × 6 of the covariance matrix.
- 0,1,4,5 Do not print the remaining tridiagonal of the covariance matrix which includes those elements in rows 7 through 51 from column one, out to and including the diagonal.
- 0,1,2,3 Do not print the eigen values and eigen vectors associated with the covariance matrix.
- 7 Print everything.

It is seen that, by a suitable choice of IQ, any combination of the three items listed above can be printed out. The default value of IQ is zero.

IS — A flag to determine the coordinate system in which the state vector, the covariance matrix, and the eigen vectors will be printed out. It can have integral values from 0 through 4 with the following meanings:

- ≤ 1 Printout is in earth-centered inertial coordinates.
- 2 Printout is in in-plane inertial coordinates.

3 Printout is in earth-centered rotating coordinates.

4 Printout is in in-plane rotating coordinates.

The default value of IS is zero.

DX(I) A 51-element array comprising the initial values of the estimated state vector, where the first six associated with the missile's position and velocity are expressed in the coordinate system specified by INPX, and the remaining 45 associated with the IMU are independent of the coordinate system. The default values for all of the DX's are zero.

DATQ(I) A 51-element array specifying the initial diagonal elements of the covariance matrix that are the variances of the state variables expressed in the coordinate system specified by INPQ. This array is input only when the value of INPQ is less than zero. The default value of DATQ(I) is zero.

Q(I,J) A 51 × 51 array containing the initial elements of the covariance matrix that are entered only when INPQ is greater than zero. If Q(I,J) is entered when INPQ is less than zero, its diagonal elements will be superseded by the values of DATQ. The default values for all of the elements of Q(I,J) are zero.

IMODE A flag to determine which mode of operation of the program is in use, as described in Sec. III on Program Options. It can have the values 0, 1, or 2 with the following meanings:

0 The program operates in Mode 0, in which covariance is propagated along the trajectory and updated by the Kalman filter.

1 The program operates in Mode 1, in which observational data are generated and recorded for later use in Mode 2.

2 The program operates in Mode 2, in which a determination of the state vector is made using

observational data generated either by Mode 1 or from the tracking of an actual missile.

Namelist/TRNFIN/

IOBS — A flag to control the printout of the tracker observational data, which include: the tracker identification numbers; its latitude, longitude, and altitude; its path elevation angle, azimuth angle, and speed; and, in addition, the magnitude of the measured quantity and the number of that type of measurement that has been made by that tracker. This flag is an integer with the following interpretations:

- 0 Do not print observational information.
- >0 Print observational data only at the interval specified by PRNTDT. Thus, if the observation interval is less than the print interval, only those observations at the specified printout time are printed.
- <0 Observational information for all measurements is printed out regardless of the print interval.

The default value of IOBS is zero.

IR — A flag to control the printout of the standard deviations of the measurements. It has an integer value which is interpreted as follows:

- 0 Do not print standard deviations of measurements.
- ≠0 Print standard deviations of measurements.

The default value of IR is zero.

IH — A flag to control the printout of the H matrix generated by the measurements. It is an integer which is interpreted as follows:

- 0 Do not print the H matrix.
- ≠0 Print the H matrix.

The default value of IH is zero.

<u>TIN</u> The starting time of the run, in seconds; it is only entered in the first stage since it is automatically set to its appropriate value in succeeding stages. The default value of TIN is zero.

<u>IRND</u> A flag to determine whether random variations are introduced into the tracker measurements. If IRND is zero, no such variations are included; if it is not zero, then a random variation is selected from a gaussian distribution whose standard deviation is that specified for the given type of measurement. This variation is then added to the already biased value of the measurement. The default value of IRND is zero.

<u>GHA</u> The initial value of the Greenwich Hour Angle, which is the longitude difference between Greenwich and the first point of Aries and is expressed in degrees. It is specified only for the first stage, since it is automatically reset to its appropriate value at the beginning of each succeeding stage. The default value of GHA is zero.

<u>TMAX</u> The termination time of this stage of the flight, measured in seconds from the start of the flight. If this stage is intended to terminate at impact, the value of TMAX should be made larger than the estimated time to insure that the missile actually reaches impact. The default value of TMAX is zero.

<u>ICOMP</u> A flag to determine whether gravity compensation is used in the IMU, as described in the sections on gyroscope and accelerometer errors. If ICOMP is a nonzero value, compensation is used. The default value of ICOMP is zero.

<u>DELMN</u> The minimum elevation angle of a satellite tracker relative to the user vehicle such that the tracker can be seen by the user. The default value of DELMN is zero.

HMIN If a minimum altitude exists along the path connecting a satellite tracker and a user vehicle, this minimum cannot be less than HMIN if tracking is to be accomplished. HMIN is expressed in feet and has a default value of zero.

Namelist/MODIN/

DTP The maximum allowable integration interval, in seconds.

IPRNT A flag to control the printout of the position and velocity of the missile. If its value is less than zero, the position and velocity are printed out at the end of each integration interval; if IPRNT is greater than zero, these data are printed out only at the specified print time. The absolute value of IPRNT determines the coordinate system and format of the position and velocity printout as follows:

0 Do not print missile position and velocity data.

1 Print missile position and velocity relative to an earth-centered rotating coordinate system as x_R, y_R, z_R, $\dot{x}_R$, $\dot{y}_R$, and $\dot{z}_R$, measured in feet and feet per second.

2 Print missile position and velocity relative to an in-plane rotating coordinate system, as latitude, longitude, altitude (in feet), path elevation angle (in degrees), path azimuth angle (in degrees), and velocity (in feet per second).

3 Print missile position and velocity relative to an earth-centered inertial coordinate system as x_I, y_I, z_I, $\dot{x}_I$, $\dot{y}_I$, and $\dot{z}_I$, in feet and feet per second.

The default value of IPRNT is zero.

IP — A flag to determine the coordinate system in which the initial position and velocity of the missile are expressed, with the following options:

1 Earth-centered rotating coordinate system.

2 Rotating topographical latitude and longitude coordinate system.

3 Earth-centered inertial coordinate system.

4 Inertial topographical latitude and longitude coordinate system.

There is no default value of IP and it is entered only for the first stage.

POS(I) — A three-element array that specifies the initial position of the missile; however, its form depends on the value of IP, defined above.

- If IP equals one, the three elements of POS are the cartesian coordinates of the missile's position, in feet, relative to an earth-centered rotating coordinate system.
- If IP equals two, the three elements of POS are the latitude (in degrees), the longitude (in degrees), and the altitude (in feet) of the missile relative to the rotating earth.
- If IP equals three, the three elements of POS are the three cartesian coordinates, in feet, of the missile relative to an earth-centered inertial coordinate system.
- If IP equals four, the three elements of POS are the latitude (in degrees), the longitude (in degrees), and the altitude (in feet) of the missile relative to a nonrotating earth.

There is no default value for POS and it should only be entered in the first stage of the flight.

VEL(I) — A three-element array that specifies the initial velocity of the missile although, as in the case of the POS array, its form depends on the value of IP, as follows:

- If IP equals one, the three elements of VEL are the cartesian components of the missile's velocity, in feet per second, relative to the earth-centered rotating coordinate system.
- If IP equals two, the three elements of VEL are the initial path elevation angle (in degrees), the path azimuth angle (in degrees), and the velocity (in feet per second) of the missile relative to the rotating earth.
- If IP equals three, the three elements of VEL are the cartesian components of the initial missile velocity, in feet per second, relative to the earth-centered inertial coordinate system.
- If IP equals four, the three elements of VEL are the path elevation angle (in degrees), the path azimuth angle (in degrees), and the velocity (in feet per second) relative to a nonrotating earth.

There is no default value for VEL and it should only be entered for the first stage of the flight.

<u>TO</u> The time for which the values of POS and VEL are defined; it is measured in seconds relative to the start of the run and has no default value. It should only be specified for the first stage of the flight.

<u>TDOT(I)</u> A 30-element array that represents the values of time at which the three components of acceleration are specified when a tabular acceleration input is used. There is no default value for TDOT and it should only be entered if IBST is greater than zero. The final value of TDOT should be the same as TMAX of the stage to avoid unspecified values of input acceleration.

<u>DDOT(I,J)</u> A 30 × 3 array where the three elements of the *Ith* row are the three components of missile acceleration, in feet per second^2, at time TDOT(I). These components

are in launch-centered inertial coordinates or launch-centered rotating coordinates, depending on whether IBST is equal to one or two. There is no default value for DDOT and it should only be entered if IBST is greater than zero.

PPSI — The launch azimuth, in degrees, measured clockwise from north. It has no default value and must be specified even though a path azimuth may have been specified as VEL(2) for IP equal to two or four.

IOBL — A flag to specify whether the earth is spherical or oblate, as follows:

0 Spherical earth.

1 Oblate earth.

The default value is zero.

IROT — A flag to specify whether the earth is nonrotating or rotating, as follows:

0 Nonrotating earth.

1 Rotating earth.

The default value is one.

ALPHA(I) — A 6-element array representing the accelerometer and gyroscope orientation angles α_1, α_2, α_3, α_1', α_2', and α_3', as defined in the appendix. The default values are 15, -36, -36, -30, -120, and -120 deg, respectively.

BETA(I) — A 6-element array representing the accelerometer and gyroscope orientation angles β_1, β_2, β_3, β_1', β_2', and β_3'. The default values are 0, 28, -28, 0, 0, and 90 deg, respectively.

PHIA(I) — A 6-element array representing the accelerometer and gyroscope orientation angles γ_1, γ_2, γ_3, γ_1', γ_2', and γ_3'. The default values are all zero.

Namelist/TRAKIN/

All indexed variables in this namelist have a dimension of 24, since they all represent characteristics of the various trackers of which there can be a maximum of 24.

<u>ALA(I)</u> The latitude of the tracker, in degrees. If the tracker is ground-based or air-based, this is geodetic latitude; otherwise, it is geocentric latitude. The default value is zero.

<u>ALO(I)</u> The longitude of the tracker, in degrees. The default value is zero.

<u>ALT(I)</u> The altitude of the tracker, in feet. The default value is zero.

<u>S(I)</u> The speed of an airborne tracker relative to the surface of the earth, in feet per second. The default value is zero.

<u>V(I)</u> The velocity of a satellite tracker relative to inertial space, in feet per second, with a default value of zero.

<u>B(I)</u> The heading of an airborne tracker relative to the rotating earth, in degrees, clockwise from north. Its default value is zero.

<u>PSI(I)</u> The heading of a satellite tracker in inertial space, in degrees, clockwise from north. Its default value is zero.

<u>GAM(I)</u> The path elevation angle of a satellite tracker in inertial space, in degrees, with a default value of zero.

<u>TO(I)</u> The time relative to the start of the run at which the foregoing variables in this namelist apply, with a default value of zero.

<u>TRTYP(I)</u> This variable is alphanumeric and indicates the type of tracker; it can be set equal to the following:

- Ground.
- Airplane.
- Satellite.
- Shadow.

The default value for this variable is BLANK.

<u>ELMN(I)</u> The minimum elevation angle, in degrees, of the target relative to the tracker for satisfactory observation. The default value is zero.

ELMX(I) The maximum elevation angle, in degrees, of the target relative to the tracker for satisfactory observation. The default value is 100.

ONA(I) The starting time of the first observing interval for the tracker measured, in seconds, from the start of the run. The default value is -10^{20}.

OFFA(I) The terminal time of the first observing interval, with a default value of $+10^{20}$.

ONB(I) The starting time of the second observing interval, with a default value of $+10^{20}$.

OFFB(I) The terminal time of the second observing interval, with a default value of $+10^{20}$.

ONC(I) The starting time of the third observing interval, with a default value of $+10^{20}$.

OFFC(I) The terminal time of the third observing interval, with a default value of $+10^{20}$.

OND(I) The starting time of the fourth observing interval, with a default value of 10^{20}.

OFFD(I) The terminal time of the fourth observing interval, with a default value of 10^{20}.

DELT(I) The interval, in seconds, between observation times for the tracker. The default value is zero.

D1(I) The constant C_1, in feet, in the expression for the standard deviation in the range measurement, Eq. (34).

D2(I) The constant C_2, in feet per second, in the expression for the standard deviation in the range rate measurement, Eq. (40).

D3(I) The constant C_3, in milliradians, in the expression for the standard deviation of the elevation angle measurement, Eq. (44).

D4(I) The constant C_4, in milliradians, in the expression for the standard deviation in the azimuth angle measurement, Eq. (48).

D5(I) The constant C_5, in feet, in the expression for the standard deviation in the hyperbolic range measurement, Eq. (52).

D6(I) The constant C_6, in feet per second, in the expression for the standard deviation in the hyperbolic range rate expression, Eq. (55).

D7(I) The constant C_7 in Eq. (34).*

D8(I) The constant C_8 in Eq. (40).

D9(I) The constant C_9 in Eq. (44).

D11(I) The constant C_{11} in Eq. (52).

D12(I) The constant C_{12} in Eq. (55).

D13(I) The constant C_{13} in Eq. (34).

D14(I) The constant C_{14} in Eq. (40).

D15(I) The constant C_{15} in Eq. (44).

D16(I) The constant C_{16} in Eq. (48).

D17(I) The constant C_{17} in Eq. (52).

D18(I) The constant C_{18} in Eq. (55).

D19(I) The constant C_{19} in Eq. (34).

D20(I) The constant C_{20} in Eq. (40).

D21(I) The constant C_{21} in Eq. (44).

D22(I) The constant C_{22} in Eq. (48).

D23(I) The constant C_{23} in Eq. (52).

D24(I) The constant C_{24} in Eq. (55).

DRANG(I) The bias** in the range measurement, in feet.

DRRT(I) The bias in the range measurement, in feet per second.

DELEV(I) The bias in the elevation angle measurement, in milliradians.

DAZIM(I) The bias in the azimuth angle measurement, in milliradians.

DRNG(I) The tropospheric range measurement bias, in feet, for a 90-deg elevation angle.

---

*The default values for the constants D1 through D24 are zero; however, it is important that the standard deviations of the measurements as given by Eqs. (34), (40), (44), (48), (52), and (55) should be nonzero for each type of measurement made by a given tracker. This can be accomplished by assigning nonzero values to the appropriate constants in these expressions.

**The 11 bias terms listed above have default values of zero, and, if these biases are entered in the namelist, they supersede any values that may have been previously selected from gaussian distributions.

DRAT(I) The tropospheric range rate measurement bias, in feet per second, for a 90-deg elevation angle.

DELV(I) The tropospheric elevation angle measurement bias, in milliradians, for a 90-deg elevation angle.

DLAT(I) The bias in the tracker latitude, in feet.

DLON(I) The bias in the tracker longitude, in feet.

DALT(I) The bias in the tracker altitude, in feet.

DVEL(I) The bias in the tracker velocity, in feet.

NTR(I) The tracker identification number associated with the tracker.

AZIM(I) If set equal to TRUE, the tracker can measure azimuth angle.*

ELEV(I) If set equal to TRUE, the tracker can measure elevation angle.

RANGE(I) If set equal to TRUE, the tracker can measure range.

RRT(I) If set equal to TRUE, the tracker can measure range rate.

HRANGE(I) If set equal to TRUE, the tracker can measure hyperbolic range.

HRRT(I) If set equal to TRUE, the tracker can measure hyperbolic range rate.

IPSMD A flag to determine the part cular mode of hyperbolic range measurement, which is to be used as follows:

0 The minimum range tracker is used as the reference at each observation time.

1 The minimum range tracker data from the previous measuring time are propagated forward using the state transition matrix, to be used as a reference at the present observation time.

2 The previous measurement data for each tracker are propagated forward by the state transition matrix, to be used as references for the

*The six logical variables all have default values of FALSE.

respective trackers at the current observation time (delta range option).

The default value is zero.

IPRMD A flag to determine the particular mode of hyperbolic range rate measurement, which is to be used as follows:

0 The minimum range tracker is used as a reference at each observation time.

1 The minimum range tracker data at the previous observation time are propagated forward, using the state transition matrix, to be used as a reference at the current observation time.

Multistage Operation

The organization of the input deck described above is that required for the first stage of a missile's flight. If, as is usually the case, there is more than one stage in the flight, the organization of the deck is as follows:

```
Stage 1    & BIASIN
               Unformatted data cards
           & END
           FORMATTED DATA CARD (ISTG=1)
           & STATIN
               Unformatted data cards
           & END
           & TRNFIN
               Unformatted data cards
           & END
           & MODIN
               Unformatted data cards
           & END
           & TRAKIN
               Unformatted data cards
           & END
```

Stage 2 FORMATTED DATA CARD (ISTG=0)

& STATIN

Unformatted data cards

& END

& TRNFIN

Unformatted data cards

& MODIN

Unformatted data cards

& END

& TRAKIN

Unformatted data cards

& END

The example above represents only two stages, but the same operation could be repeated for any number of stages, with ISTG set equal to unity in all but the last stage. If an input variable is not specified in its namelist, it will retain its value from the previous stage, or its default value if the input is for the first stage. As is frequently the case, there may be no changes in the values of the variables in a given namelist in a succeeding stage. Nevertheless, it is still necessary to include the namelist title card and END card in the deck, even though there are no data cards between them.

Multiple Runs

It is also possible to submit multiple missile flights in sequence by repeating the same deck organization shown above for the multistage case. The only difference is that the variable IRUN is not specified in BIASIN after the first run.

It should be noted that values of the input variables do not carry over from one run to the next, but revert to their default values unless entered again.

Block Data

In addition to the input values described above, there are certain parameters which are set within the program in the block data subprogram. These quantities are as follows:

Degrees per radian

DEG = 57.2957795131.

Earth gravitational constant

HMU = 1.407645×10^{16} ft^3/sec^2.

Earth angular rate

OMEG = $7.29211585 \times 10^{-5}$ rad/sec.

Earth radius

AE = 20,925,696 ft.

Earth eccentricity

EPS = 0.082.

OUTPUT FORMAT

Title Page

On this page, the program name and a brief statement of the operations performed by the four major subroutines are printed.

Input Parameters and Comments

In this section, the input data, as well as any initial conditions generated from this input data, are printed out in the following order at the beginning of each stage of the flight.

1. The value of the print interval, in seconds.
2. The values of the flags ISTG, IBST, IMODE, INPQ, INPX, IX, IQ, IS, together with the interpretation of these numerical assignments.
3. If the covariance matrix is entered as diagonal elements only (INPQ < 0), these diagonal elements are printed out.
4. The values of the initial biases of the 51 state variables (DXC), as selected from gaussian distributions, or their default values of zero. These values are only printed at the beginning of the first stage.

5. The initial and terminal times, in seconds, for this stage.
6. The value of the Greenwich Hour Angle, in degrees, at the beginning of the stage.
7. The values of the flags IOBS, IR, IH, IRND, together with the interpretation of these numerical assignments.
8. The initial position of the missile and the coordinate system in which it is expressed. If IP equals 2 or 4, the position is given in latitude, longitude, and altitude; if IP equals 1 or 3, the position is in cartesian coordinates.
9. The initial missile velocity in the same coordinate system as the position. If IP equals 2 or 4, the velocity is given in terms of its path elevation angle, path azimuth angle, and velocity magnitude; if IP equals 1 or 3, it is given in cartesian coordinates. These initial position and velocity values are printed out only at the beginning of the first stage.
10. The time TO at which the initial conditions apply, measured in seconds, from the start of the run.
11. The maximum value of the integration interval DTP, in seconds.
12. The variances of state variables associated with the IMU.
13. The tabular acceleration profile, giving the three components of missile acceleration as a function of time, if IBST does not equal zero.
14. A list of which trackers are available to make measurements during this stage, where they are based, and what they can measure.
15. The initial position and velocity of each tracker and the times at which these conditions exist.
16. The limits of the intervals during which the trackers can observe, the interval between observations, and the maximum and minimum elevation angles of the trackers.
17. The measurement biases in range, range rate, elevation angle, and azimuth angle for each tracker.
18. The biases in latitude, longitude, altitude, and speed of each tracker.

19. The tropospheric biases in range, range rate, and elevation angle for an elevation angle of 90 deg.
20. The 24 constants used to express the standard deviations of the measurements for each tracker.

Output Legends

This page is printed out at the beginning of each stage and gives the format of the data which will be printed out during this stage. The format is determined by the flags IQ, IS, IX, IOBS, IR, IH, and IPRNT.

Statistical Estimation

This information is printed out at the end of each print interval and includes the following items:

1. The time for which these data apply, measured in seconds, from the start of the run.
2. The position and velocity of the missile, expressed in the coordinate system, determined by the value of IPRNT.
3. The first six elements of the actual state vector DXC that give the deviations, expressed in the coordinate system determined by the flag IS, of the missile's position and velocity from the nominal trajectory.
4. The elements of the estimated state vector DX, in the same coordinate system as DXC. If no external measurements have been made since the last print time, only the first six elements of DX are printed for comparison with the corresponding DXC values. If measurements have been made, the remaining 45 elements of DX associated with the IMU parameters are also printed out for comparison with their actual values listed in the section on input parameters and comments.
5. The values of the first six elements of the state variable X that give the current estimated position and velocity of the missile in the coordinate system specified by IS. If the program is being used in the simulation mode (IMODE=2), then

the remaining 45 state variables, the estimated IMU parameters, are also printed out. These values of X, as well as those of DXC and DX, are only printed if the flag IX is nonzero.

6. The standard deviations of the estimated variations of 51 state variables that are the square roots of the diagonal elements of the state covariance matrix.
7. The upper left (6 × 6) of the state covariance matrix showing the correlation between the components of missile position and velocity. The standard deviations and this 6 × 6 matrix are expressed in the coordinate system specified by IS and are only printed if the value of the flag IQ is odd.
8. The remaining tridiagonal elements of the state covariance matrix, showing the correlation between the missile position and velocity variations, as well as the IMU parameter variations. These elements are printed if the flag IQ is equal to 2, 3, or 7.
9. The eigen values and eigen vectors associated with the position covariance matrix are print in a six-column format, in which the third column is the eigen value and the fourth through sixth columns are the three components of the associated eigen vector. The first column represents the semiaxis of the 50 percent confidence ellipsoid, and the second column is the semiaxis of the 95 percent confidence ellipsoid for this same vector direction.
10. The eigen values and eigen vectors associated with the velocity covariance matrix are printed out in the same format. The position and velocity eigen value data are only printed when the flag IQ is greater than 4.
11. The data associated with observations by the external trackers start with a printout of the observation time, followed by the observational data of each of the trackers. If a tracker makes no observation, the printout is a single line, starting with the tracker identification number and followed by "NONE" and six zeros. If a tracker does make an observation, the

output consists of, at most, four lines. The first line starts with the tracker identification number and is followed by the latitude, longitude, and altitude of the tracker, as well as its path elevation angle, path azimuth angle, and speed. These values are in the same columns as the six zeros in the case of the nonobserving tracker. The second line gives the elevation angle and range of the target relative to the tracker. The third line lists the nominal value of any measurement made by this tracker in columns between those used for the position and velocity data. The order of the measurements in these columns is range, range rate, azimuth angle, elevation angle, hyperbolic range, and hyperbolic range rate.

Alongside of each measurement value is printed the total number of this type of measurement made by this tracker up to this time. If IPSMD or IPRMD is zero, then the minimum range tracker is indicated in the column for hyperbolic range or hyperbolic range rate. If IPSMD or IPRMD is unity, the minimum range tracker is indicated by an M beside the number of hyperbolic range or hyperbolic range rate measurements for that tracker, indicating that these are the tracker data that are propagated forward as a reference.

The fourth line of the printout for a given tracker occurs only for the simulation mode of operation (IMODE=2) and gives the residual differences between the measurement and its nominal value as the reference trajectory is progressively modified. These values are printed in the same columns as the corresponding measurements.

12. The elements of the H matrix are printed in a six-column format with each row corresponding to one measurement, starting with the first measurement by the lowest numbered tracker and ending with the last measurement by the highest numbered tracker. These data are printed out only if the flag IH is nonzero.

13. The standard deviations of the various measurements are printed in a four-column format with one value for each measurement so that measurements 1 through 4 are in the first row, 5 through 8 in the second, and so on--through the final measurement.

Expanded Printout

If the print interval PRNTDT is larger than the interval between tracker observations, the tracker observations and the corresponding missile position and velocity data at these intermediate observation times will not be printed out. If this information is required, it can be obtained by means of the flags IPRNT and IOBS. If IPRNT is negative, the missile position and velocity data at each observation time are printed out in the same form as described in the previous section on statistical estimation. By making IOBS negative, the tracking data at each observation time are printed out immediately following the corresponding missile position data in the same format described previously.

At the specified printout time, since the missile position and velocity data have already been printed, only the values of DXC, DX, X, the standard deviations of the estimates, the state covariance matrix, the eigen values, and the eigen vectors are printed out in the statistical estimation format.

Flight Termination

The final statistical estimation occurs at a time equal to either TMAX for the final stage or at the time of impact, whichever is smaller. If the run is terminated at TMAX, the statistical estimation is in the same format as that described above. However, if the run is terminated by impact with earth, the following additional information is printed:

1. A statement indicating that the vehicle is down.
2. The semiaxes, in feet, of the error ellipse on the ground and the orientation of its major axis relative to an in-plane rotating coordinate system.

3. The major and minor axes of the 50 percent and 95 percent confidence ellipses, in feet.
4. The down range and cross range standard deviations, in feet.
5. The down range and cross range bias errors in the estimated impact point, in feet.

Appendix

COORDINATE SYSTEMS

The various coordinate systems used in the formulation of this program are described below, together with the matrix transformation relating them.

EARTH-CENTERED INERTIAL (ECI)

ECI is the basic coordinate system, in which all computations are performed. Its origin is at the center of the earth with its z axis colinear with the earth's spin axis and its x axis in the direction of the first point of Aries.

EARTH-CENTERED ROTATING (ECR)

The ECR system al.) has its origin at the earth's center and its z axis along the earth's spin axis; its x axis is in the plane of the Greenwich meridian. The relation between the ECR and ECI systems is shown in Fig. 1, where Γ is the Greenwich Hour Angle between the plane of the Greenwich meridian and the first point of Aries, given by the relation

$$\Gamma = \Gamma_0 + \Omega_E(t - t_0) \ , \qquad \text{(A-1)}$$

where Γ_0 is the initial value of Γ at some reference time t_0, and Ω_E is the spin angular rate of the earth.

The transformation of a vector from ECI to ECR coordinates is given by the matrix equation

$$\overline{R}^{(ECR)} = \left| A_{ECR} \right| \overline{R}^{(ECI)} \ , \qquad \text{(A-2)}$$

where the transformation matrix is given by

$$\left| A_{ECR} \right| = \begin{vmatrix} \cos\Gamma & \sin\Gamma & 0 \\ -\sin\Gamma & \cos\Gamma & 0 \\ 0 & 0 & 1 \end{vmatrix} \ . \qquad \text{(A-3)}$$

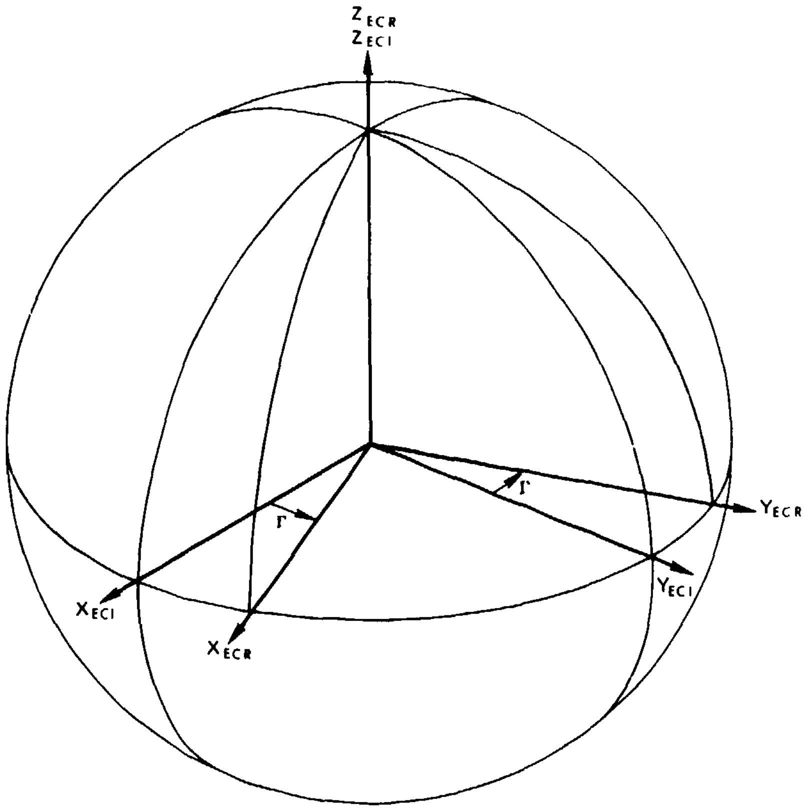

Fig. 1 — Earth centered rotating coordinate system

LAUNCH-CENTERED ROTATING (LCR)

The LCR system has its origin at the launch site of the missile with its z axis vertical and its x axis in the direction of the launch azimuth. The relationship of this system to the ECI system is shown in Fig. 2, where λ_0, β_0, and ψ_0 are the latitude, longitude, and path azimuth, respectively, at the launch site. The transformation between ECI and LCR is given by the relation

$$\bar{R}^{(LCR)} = \left| A_{LCR} \right| \bar{R}^{(ECI)} , \tag{A-4}$$

where the transformation matrix has the form

$$
\left|A_{LCR}\right| = \left|\begin{matrix}
\begin{pmatrix} -\sin\psi_0 \sin(\beta_0 + \Gamma) \\ -\cos\psi_0 \sin\lambda_0 \cos(\beta_0 + \Gamma) \end{pmatrix} & \begin{pmatrix} \sin\psi_0 \cos(\beta_0 + \Gamma) \\ -\cos\psi_0 \sin\lambda_0 \sin(\beta_0 + \Gamma) \end{pmatrix} & \begin{pmatrix} \cos\psi_0 \cos\lambda_0 \end{pmatrix} \\
\begin{pmatrix} \cos\psi_0 \sin(\beta_0 + \Gamma) \\ -\sin\psi_0 \sin\lambda_0 \cos(\beta_0 + \Gamma) \end{pmatrix} & \begin{pmatrix} -\cos\psi_0 \cos(\beta_0 + \Gamma) \\ -\sin\psi_0 \sin\lambda_0 \sin(\beta_0 + \Gamma) \end{pmatrix} & \begin{pmatrix} \sin\psi_0 \cos\lambda_0 \end{pmatrix} \\
\begin{pmatrix} \cos\lambda_0 \cos(\beta_0 + \Gamma) \end{pmatrix} & \begin{pmatrix} \cos\lambda_0 \sin(\beta_0 + \Gamma) \end{pmatrix} & \begin{pmatrix} \sin\lambda_0 \end{pmatrix}
\end{matrix}\right| . \qquad \text{(A-5)}
$$

Since Γ is a function of time through Eq. (A-1), the transformation matrix also changes with time as the launch site rotates about the earth's axis.

LAUNCH-CENTERED INERTIAL (LCI)

The LCI system also has its origin at the launch site with its z axis in the direction of the vertical at the time of launch and its x axis in the direction of the inertial launch azimuth. The

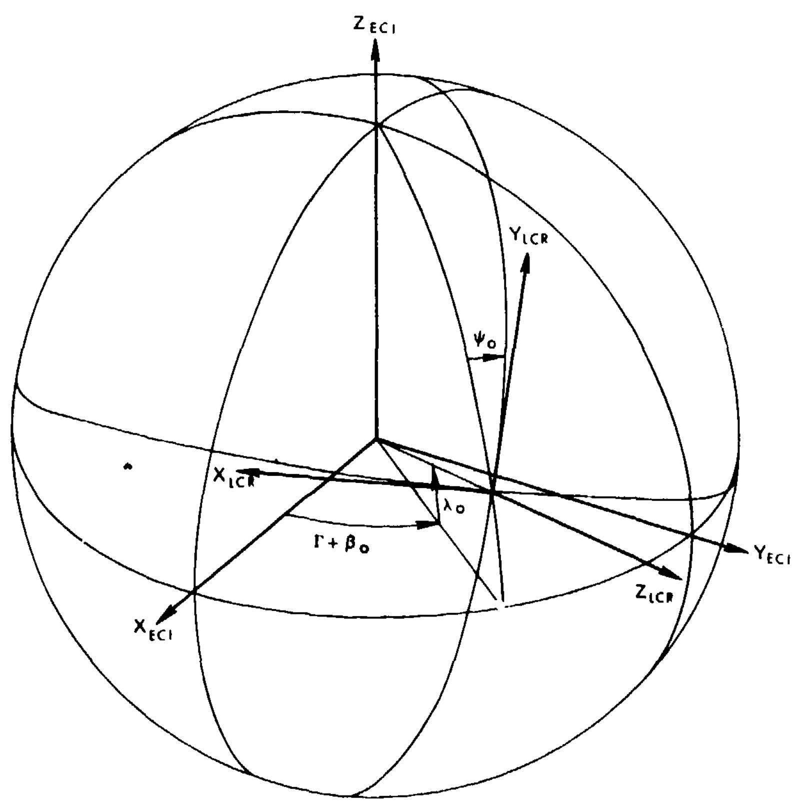

Fig. 2 — Launch centered rotating coordinate system

transformation from ECI to LCI is given by

$$\bar{R}^{(LCI)} = \left|A_{LCI}\right| \bar{R}^{(ECI)} , \tag{A-6}$$

where the transformation matrix $\left|A_{LCI}\right|$ has the same form as $\left|A_{LCR}\right|$ in Eq. (A-5), except that Γ is replaced by its initial value Γ_0. Thus, $\left|A_{LCI}\right|$ is independent of time and the LCI coordinate system maintains a constant orientation relative to the ECI system.

IN-PLANE ROTATING (IPR)

The IPR system has its origin on the missile or user vehicle, with its z axis along the local vertical and its x,z plane containing the missile velocity vector relative to the rotating earth. The relation of this system to the ECR and ECI systems is shown in Fig. 3. The

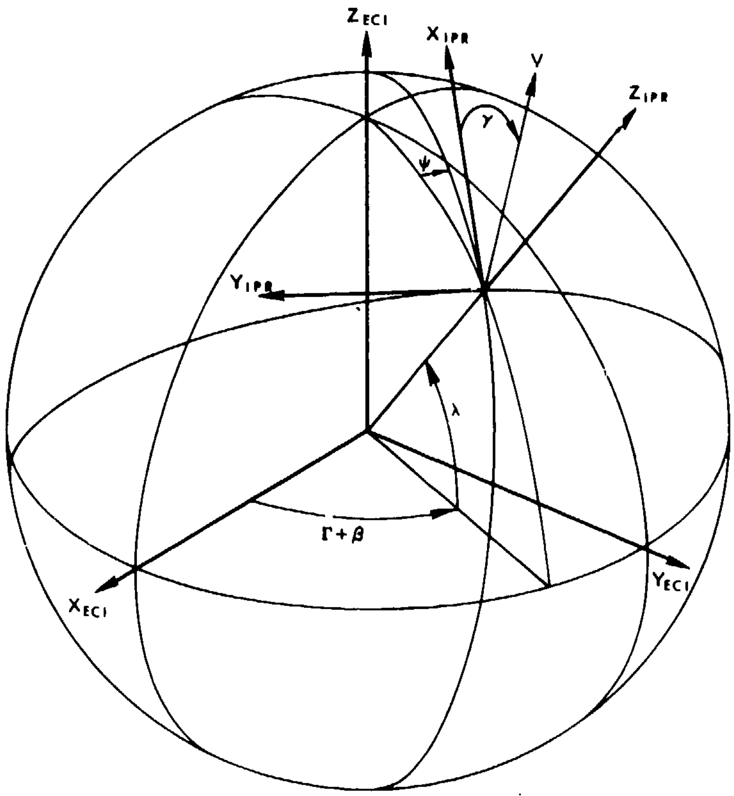

Fig. 3 — In-plane rotating coordinate system

transformation from ECI to IPR is given by

$$\bar{R}^{(IPR)} = \left|A_{IPR}\right| \bar{R}^{(ECI)} , \tag{A-7}$$

where the transformation matrix $\left|A_{IPR}\right|$ has the same form as that for $\left|A_{LCR}\right|$ in Eq. (A-5), with the exception that the quantities λ_0, β_0, and ψ_0 are replaced by their values at the missile's present position on its flight path.

IN-PLANE INERTIAL (IPI)

The IPI system differs from the IPR system only in that the x,z plane is oriented to contain the velocity vector relative to inertial space. The transformation from ECI to IPI is given by

$$\bar{R}^{(IPI)} = \left|A_{IPI}\right| \bar{R}^{(ECI)} , \tag{A-8}$$

where $\left|A_{IPI}\right|$ has the same form as $\left|A_{IPR}\right|$ except that the value of the path azimuth ψ refers to the inertial velocity vector.*

PLATFORM (P)

This system is associated with the stabilized platform on which the inertial instruments are mounted. Ideally, the platform coordinate system has the same orientation as the LCI or LCR system, depending on the particular guidance system used. Thus, the transformation from ECI to P is given by

$$\bar{R}^{(P)} = \left|A_{P}\right| \bar{R}^{(ECI)} , \tag{A-9}$$

---

*It should be noted that the IPI system is not actually an inertial reference system, since its orientation does change relative to inertial space although the inertial velocity remains in the x,z plane.

where

$$\left|A_P\right| = \left|A_{LCI}\right| \quad \text{or} \quad \left|A_{LCR}\right| . \tag{A-10}$$

MISALIGNED PLATFORM (P_m)

The actual platform has small angular misalignments relative to the idealized platform such that the transformation from P to P_m is given by

$$\bar{R}^{(P_m)} = \left|A_{P_m}\right| \bar{R}^{(P)} , \tag{A-11}$$

where the transformation matrix is given by

$$\left|A_{P_m}\right| = \begin{vmatrix} 1 & K_2 & -K_1 \\ -K_2 & 1 & K_3 \\ K_1 & -K_3 & 1 \end{vmatrix} \tag{A-12}$$

and K_1 is a small rotation about the y_P axis, K_2 about the z_P axis, and K_3 about the x_P axis.

ACCELEROMETER

This system is associated with the *ith* accelerometer with its x axis along the input axis, y along the cross axis, and z along the normal axis. Its orientation relative to the P system is shown in Fig. 4, and involves a rotation through an angle α_i about the y axis, followed by a rotation β_i about the z axis, and finally a rotation of γ_i about the x axis. The resulting transformation from P to a_i is given by

$$\bar{R}^{(a_i)} = \left|A_{a_i}\right| \bar{R}^{(P)} , \tag{A-13}$$

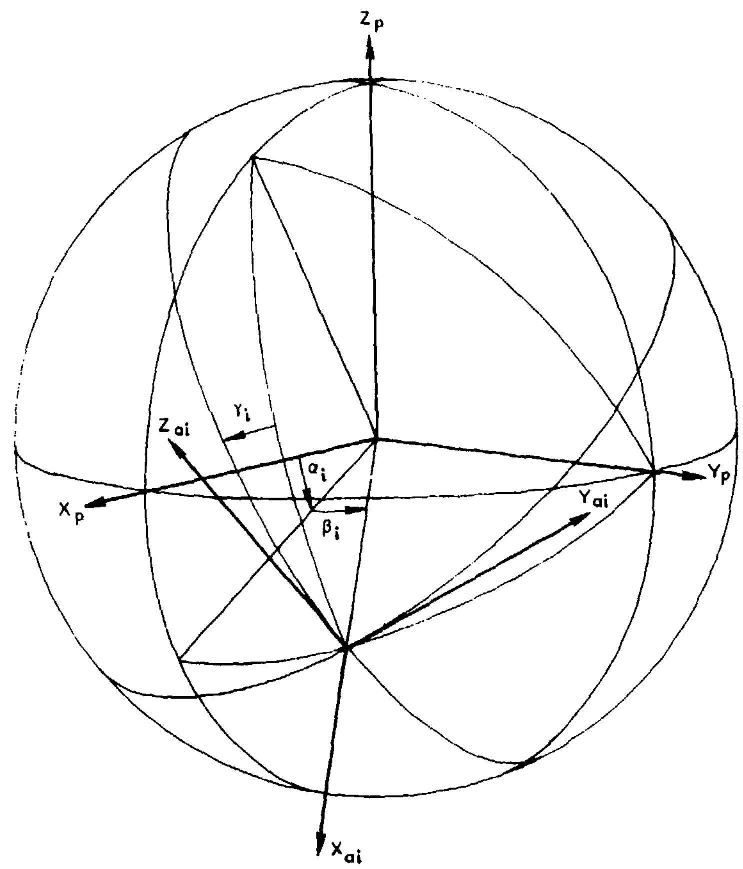

Fig. 4—Accelerometer coordinate system

where the transformation matrix is given by

$$\left|A_{a_i}\right| = \begin{vmatrix} \left(\cos\alpha_i \cos\beta_i\right) & \left(\sin\beta_i\right) & \left(-\sin\alpha_i \cos\beta_i\right) \\ \begin{pmatrix}\sin\alpha_i \sin\gamma_i \\ -\cos\alpha_i \sin\beta_i \cos\gamma_i\end{pmatrix} & \left(\cos\beta_i \cos\gamma_i\right) & \begin{pmatrix}\cos\alpha_i \sin\gamma_i \\ +\sin\alpha_i \sin\beta_i \cos\gamma_i\end{pmatrix} \\ \begin{pmatrix}\sin\alpha_i \cos\gamma_i \\ +\cos\alpha_i \sin\beta_i \sin\gamma_i\end{pmatrix} & \left(-\cos\beta_i \sin\gamma_i\right) & \begin{pmatrix}\cos\alpha_i \cos\gamma_i \\ -\sin\alpha_i \sin\beta_i \sin\gamma_i\end{pmatrix} \end{vmatrix} . \qquad \text{(A-14)}$$

<u>GYROSCOPE</u> (g_i)

The g_i system is associated with the *ith* gyroscope and has its x axis along the input axis, y along the output axis, and z along the spin axis. As in the case of the accelerometer axes, the orientation of g_i relative to the platform axes involves rotations of α_i', β_i', and

γ_i' about the y, z, and x axes, respectively. The resulting transformation is given by

$$\bar{R}^{(g_i)} = \left|A_{g_i}\right| \bar{R}^{(P)} , \tag{A-15}$$

where the transformation matrix $\left|A_{g_i}\right|$ is of the same form as $\left|A_{a_i}\right|$ in Eq. (A-14), but using the primed variables defined above.

TRACKER (T_i)

The T_i system is associated with the *ith* tracker and has its origin at the tracker position with its x axis to the east, y axis north, and z axis vertical. The orientation of T_i relative to ECR is shown in Fig. 5, where λ_{T_i} is the tracker latitude, and β_{T_i} the

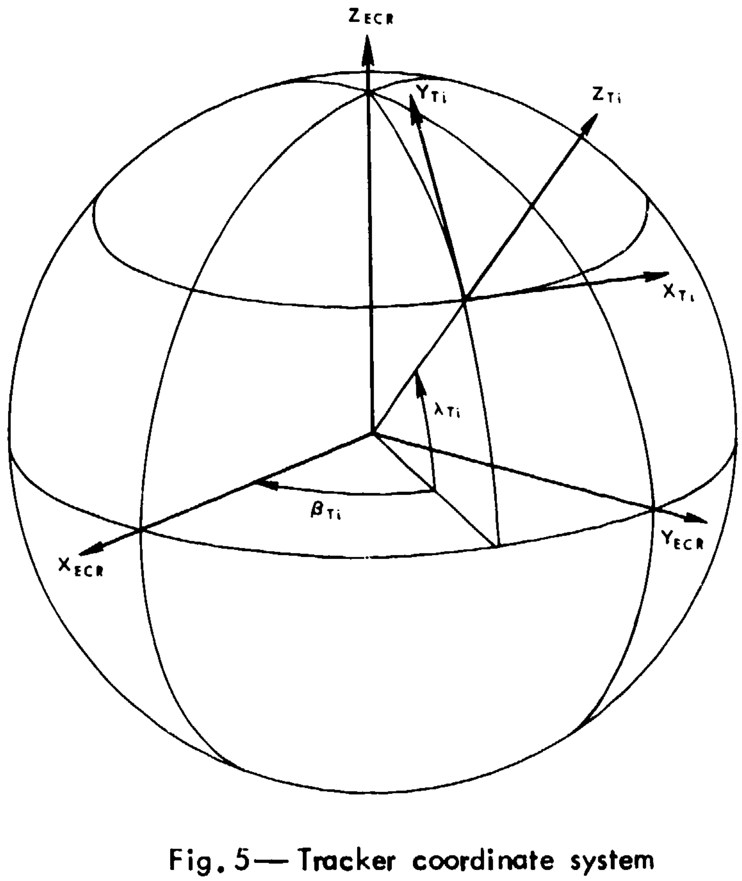

Fig. 5— Tracker coordinate system

tracker longitude. The transformation from ECR to T_i is given by

$$\bar{R}^{(T_i)} = \left|A_{T_i}\right| \bar{R}^{(ECR)} , \tag{A-16}$$

where the transformation matrix has the form

$$\left|A_{T_i}\right| = \begin{vmatrix} -\sin \beta_{T_i} & \cos \beta_{T_i} & 0 \\ -\sin \lambda_{T_i} \cos \beta_{T_i} & -\sin \lambda_{T_i} \sin \beta_{T_i} & \cos \lambda_{T_i} \\ \cos \lambda_{T_i} \cos \beta_{T_i} & \cos \lambda_{T_i} \sin \beta_{T_i} & \sin \lambda_{T_i} \end{vmatrix} . \tag{A-17}$$

OTHER TRANSFORMATIONS

The transformation matrix between any two of the coordinate systems defined above can be obtained as the products of the matrices defined above and their inverses.

STATE VECTOR TRANSFORMATION (A)

The matrix [A] indicated in Eq. (39) represents the transformation of the position and velocity elements of the state vector from ECI coordinates to tracker coordinates. The derivation of this matrix is as follows. If the position vector in tracker coordinates is given as

$$\bar{R}^{(T_i)} = \begin{vmatrix} x_R \\ y_R \\ z_R \end{vmatrix} , \tag{A-18}$$

and the corresponding position vector in ECI coordinates is

$$\bar{R}^{(ECI)} = \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} , \tag{A-19}$$

then the two are related by the equation

$$\bar{R}^{(T_i)} = \left|A_o\right| \bar{R}^{(ECI)} \, , \tag{A-20}$$

where the matrix $\left|A_o\right|$ is given by

$$\left|A_o\right| = \left|A_{T_i}\right| \times \left|A_{ECR}\right| \, . \tag{A-21}$$

The corresponding relations between the velocity vectors in the two systems are obtained as follows.

If the velocity vector in tracker coordinates is given by

$$\dot{\bar{R}}^{(T_i)} = \begin{vmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{z}_R \end{vmatrix} \, , \tag{A-22}$$

and the corresponding vector in ECI coordinates is

$$\dot{\bar{R}}^{(ECI)} = \begin{vmatrix} X_4 \\ X_5 \\ X_6 \end{vmatrix} \, , \tag{A-23}$$

then the two are related by the equation

$$\dot{\bar{R}}^{(T_i)} = \left|A_o\right| \times \left[\dot{\bar{R}}^{(ECI)} - \left[\bar{\omega}^{(ECI)} \times \bar{R}^{(ECI)}\right]\right] \, , \tag{A-24}$$

where $\bar{\omega}^{(ECI)}$ represents the angular velocity of the tracker coordinate system relative to inertial space in ECI coordinates. This angular velocity is the result of the rotation of the earth and the motion of the tracker relative to the rotating earth. In ECR coordinates, this angular velocity is given by

$$\bar{\omega}^{(ECR)} = \bar{\omega}_E^{ECR} + \frac{\left[\bar{R}_T^{(ECR)} \times \dot{\bar{R}}_T^{(ECR)}\right]}{R_T^2}, \tag{A-25}$$

where $\bar{R}_T^{(ECR)}$ is the vector position of the tracker in ECR coordinates given by the expression

$$\bar{R}_T^{(ECR)} = \begin{vmatrix} x_T \\ y_T \\ z_T \end{vmatrix}, \tag{A-26}$$

and $\dot{\bar{R}}_T^{(ECR)}$ is the velocity of the tracker relative to the earth in ECR coordinates expressed as

$$\dot{\bar{R}}_T^{(ECR)} = \begin{vmatrix} \dot{x}_T \\ \dot{y}_T \\ \dot{z}_T \end{vmatrix}. \tag{A-27}$$

Equation (A-25) can be rewritten in matrix form as

$$\bar{\omega}^{(ECR)} = \begin{vmatrix} 0 \\ 0 \\ \omega_E \end{vmatrix} + \frac{1}{R_T^2} \begin{vmatrix} 0 & -z_T & y_T \\ z_T & 0 & -x_T \\ -y_T & x_T & 0 \end{vmatrix} \times \begin{vmatrix} \dot{x}_T \\ \dot{y}_T \\ \dot{z}_T \end{vmatrix}$$

$$= \begin{vmatrix} (y_T\dot{z}_T - z_T\dot{y}_T)/R_T^2 \\ (z_T\dot{x}_T - x_T\dot{z}_T)/R_T^2 \\ (x_T\dot{y}_T - y_T\dot{x}_T)/R_T^2 + \omega_E \end{vmatrix}. \tag{A-28}$$

This angular rate can then be transformed into ECI coordinates as

$$\bar{\omega}^{(ECI)} = \begin{vmatrix} \omega_{Ix} \\ \omega_{Iy} \\ \omega_{Iz} \end{vmatrix} = \left| A_{ECR} \right|^{-1} \bar{\omega}^{(ECR)} . \tag{A-29}$$

Equation (A-24) can now be written as

$$\dot{\bar{R}}^{(T_i)} = \left| A_o \right| \times \dot{\bar{R}}^{(ECI)} - \left| \Omega \right| \bar{R}^{ECI} , \tag{A-30}$$

where

$$\left| \Omega \right| = \begin{vmatrix} 0 & -\omega_{Iz} & \omega_{Iy} \\ \omega_{Iz} & 0 & -\omega_{Ix} \\ -\omega_{Iy} & \omega_{Ix} & 0 \end{vmatrix} . \tag{A-31}$$

Equations (A-20) and (A-30) can be combined as

$$\begin{vmatrix} \bar{R}^{(T_i)} \\ \dot{\bar{R}}^{(T_i)} \end{vmatrix} = \left| A \right| \times \begin{vmatrix} \bar{R}^{(ECI)} \\ \dot{\bar{R}}^{(ECI)} \end{vmatrix} , \tag{A-32}$$

where $|A|$ is the desired transformation matrix given by the relation

$$\left| A \right| = \begin{vmatrix} \left| A_o \right| & & \left| 0 \right| \\ -\left| A_o \right| & \left| \Omega \right| & \left| A_o \right| \end{vmatrix} . \tag{A-33}$$

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Operating instructions for a computer program designed to provide a best estimate of the accuracy with which the motion of a vehicle can be determined on the basis of either inertial guidance measurements or external tracking, or both. The performance of the moving vehicle is described by the estimated deviation from its nominal performance. The program, developed by and used at Rand, should prove useful in the fields of ballistic missile accuracy, range instrumentation, and navigational satellites. (JDD)